

Advanced Series in Electrical and Computer Engineering – Vol. 18

BROADBAND MATCHING

THEORY AND IMPLEMENTATIONS

Third Edition



WAI-KAI CHEN

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BROADBAND MATCHING

THEORY AND IMPLEMENTATIONS

Third Edition

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THEORY AND IMPLEMENTATIONS

Third Edition

WAI-KAI CHEN

University of Illinois at Chicago, USA

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(3rd Edition)

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To Shiao-Ling and Jerome and Melissa

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Preface to the 3rd Edition

OVER the years, the fundamentals of broadband matching have evolved to include a wide range of topics and a broad range of practice. To encompass such a wide range of knowledge, the book focuses on the key concepts, models, and equations that enable the electrical engineer to analyze, design, and predict the behavior of broadband circuits. While design formulas and tables are listed, emphasis is placed on the key concepts and theories underlying the applications. The purpose of the book is to provide in a single volume a comprehensive reference work covering the broad spectrum of mathematics for circuits and filters; circuits configurations, devices, and their models. The book is written and developed for the practicing electrical engineers in industry, government, and academia. The goal is to provide the most up-to-date information in the field.

The book stresses fundamental theory behind professional applications. In order to do so, it is reinforced with frequent examples. The reader is assumed to have a certain degree of sophistication and experience. However, brief reviews of theories, principles, and mathematics of some subject areas are given. These reviews have been done concisely with perception.

The third edition presents a unified, up-to-date, and detailed account of broadband matching theory and its applications to the design of broadband matching networks, multiplexers, and amplifiers. A special feature is the addition of results that are of direct practical value. They include design curves, tables, and explicit formulas for designing networks having Butterworth, Chebyshev, elliptic or

maximally-flat group-delay response as well as for designing diplexer and, in general, multiplexer having these responses. The results are extremely useful as the design procedures can be reduced to simple computer routines. Several illustrative examples given at the last two new chapters are intended to demonstrate the applications to the practical design of modern filter circuits.

The compilation of this book would not have been possible without the contribution of my visiting scholars Professors Zhao-Ming Wang and Yi-Sheng Zhu and my doctoral student Dr Ji-An Gong. In fact, the last two chapter material is mostly based on their research work. I wish to thank them all.

May 27, 2014

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Preface to the 2nd Edition

THE BOOK was initially conceived as a revision of the original volume. Since then it evolved and was modified to such a great extent that more than one-third of the material is new. As a result, the title of the new edition has been changed to reflect more accurately its contents. Most of the new material appeared after the publication of the first edition in 1976, which was translated to Russian in 1979, and to Chinese in 1982.

In revising the first edition, I can think of many items that should be added. Judging from the interest of readers and the practical applications of the subject to engineers, I have decided to concentrate on areas that have received wide attention in recent years. The purpose of the new edition is to present a unified, up-to-date, and detailed account of broadband matching theory and its applications to the design of broadband matching networks and amplifiers. The book can be used as a later text in network theory as well as a reference for practicing engineers who wish to learn how the modern network theory can be applied to the design of many practical circuits. A special feature of this new edition is that results of direct practical value are included.

The new material starts in Chapter 6 with the presentation of explicit design formulas for broadband matching networks having low-pass or bandpass Butterworth or Chebyshev response of arbitrary order. The significance of these results is that they reduce many of the design procedures to simple arithmetic. Chapter 7 extends the classical single match to double match where both the source and

load impedances are frequency dependent. This is important because in many practical applications, the internal impedances of the available electronic sources are not purely resistive, especially at high frequencies for which the broadband matching theory is most needed. Finally, we present the real-frequency technique in Chapter 8 for both the single match and the double match. The method is a numerical one, and only utilizes real-frequency data of the load and/or source impedance. No model or analytic impedance function for the load and/or source is required. Nor is the equalizer topology or analytic form of the system transfer function assumed.

This edition contains a significant number of corrections that have been incorporated throughout the text. One inevitable result in adding new material is that the book has grown longer. It contains more material than can be adequately presented in a one-semester or two-quarter three hours-per-quarter course in network theory. This added flexibility will allow instructors to select subjects and sections to meet their needs and environment.

Since the publication of the first edition, many people have been kind enough to give me the benefit of their comments and suggestions, often at the expense of a very considerable amount of their time and energy. In particular, I am indebted to my graduate students, visiting scholars and those users of the book who have contributed to the improvement of this edition. Special thanks are due to Mr. Yi-Sheng Zhu of Dalian Marine College and my doctoral students Ms. Hui Tang and Mr. Qiang-Zhong Zha, who gave the new chapters a careful and critical reading and assisted me in preparing the index. Finally, I express my appreciation to my wife, Shiao-Ling, and children, Jerome and Melissa, for their patience and understanding during the preparation of the book.

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Naperville, Illinois

Preface to the 1st Edition

OVER the past two decades, we have witnessed a rapid development of solid-state technology with its apparently unending proliferation of new devices. In order to cope with this situation, a steady stream of new theory, being general and independent of devices, has emerged. One of the most significant developments is the introduction of scattering techniques to network theory. The purpose of this book is to present a unified and detailed account of this theory and its applications to the design of broadband matching networks and amplifiers. It was written primarily as a late text in network theory as well as a reference for practicing engineers who wish to learn how the modern network theory can be applied to the design of many practical circuits. The background required is the usual undergraduate basic courses in networks as well as the ability to handle matrices and functions of a complex variable.

In the book, I have attempted to extract the essence of the theory and to present those topics that are of fundamental importance and that will transcend the advent of new devices and design tools. The guiding light throughout the book has been mathematical precision. Thus, all the assertions are rigorously proved; many of these proofs are believed to be new and novel. I have tried to give a balanced treatment between the mathematical aspects and the physical postulates which motivate the work, and to present the material in a concise manner; using discussions and examples to illustrate the concepts and principles involved. The book also contains some of the personal contributions of the author that are not available elsewhere in the literature.

The scope of this book should be quite clear from a glance at the table of contents. Chapter 1 introduces many fundamental concepts related to linear, time-invariant n -port networks, defines *passivity* in terms of the universally encountered physical quantities *time* and *energy*, and reviews briefly the general characterizations of an n -port network. Its time-domain passivity conditions are then translated into the equivalent frequency-domain passivity criteria, which are to be employed to obtain the fundamental limitations on its behavior and utility. Thus, this chapter, as the title implies, may be taken as the foundation for any subsequent network study as well as for the material treated in the remainder of the book.

Chapter 2 gives a fairly complete exposition of the scattering matrix associated with an n -port network, starting from a one-port network and using the concepts from transmission-line theory. Fundamental properties of the scattering matrix and its relation to the power transmission among the ports are then derived. The results are indispensable in developing the theory of broadband matching to be treated in the last two chapters.

In seeking fundamental limitations on network or device behavior, performance criteria are often overly idealistic and are not physically realizable. To avoid this difficulty, Chapter 3 considers the approximation problem along with a discussion of the approximating functions. It is shown that the ideal low-pass brick-wall type of gain response can be approximated by three popular rational function approximation schemes: the maximally-flat (Butterworth) response, the equiripple (Chebyshev) response, and the elliptic (Cauer-parameter) response. This is followed by presenting the corresponding ladder network realizations which are attractive from an engineering viewpoint in that they are unbalanced and contain no coupling coils. Explicit formulas for element values of these ladder networks with Butterworth or Chebyshev gain characteristic are given, which reduce the design of these networks to simple arithmetic. Confining attention to the low-pass gain characteristic is not to be deemed restrictive as it may appear. This is demonstrated by considering frequency transformations that permit low-pass characteristic to be converted to a high-pass, band-pass, or band-elimination characteristic.

Using the results developed in the first three chapters, Chapter 4 treats Youla's theory of broadband matching in detail, illustrating every phase of the theory with fully worked out examples. In particular, the fundamental gain-bandwidth limitations for Bode's parallel RC load and Darlington's type-C load are established in their full generality. The extension of Youla's theory to active load impedance is taken up in Chapter 5. It is demonstrated that with suitable manipulations of the scattering parameters, the theory can be applied to the design of negative-resistance amplifiers. This is especially significant in view of the continuing development of new one-port active devices such as the tunnel diode. Many readers will find the perusal of this chapter to be a gratifying and stimulating experience.

In selecting the level of presentation, considerable attention has been given to the fact that many readers may be encountering these topics for the first time. Thus basic introductory material has been included. For example, since many readers are not familiar with the subject of elliptic functions in Chapter 3 on Approximation and Ladder Realization, an entire section is devoted to the discussion of elliptic functions and some of their fundamental properties that are needed in subsequent analysis. In fact, the section on elliptic response has never been so concisely and systematically treated elsewhere.

The text has grown out of a graduate course entitled "Linear Network Theory" organized at Ohio University. Over the period of years, the material has naturally evolved and up-dated into a shape quite different from the original. However, the basic objective of establishing the fundamentals in this area has remained unchanged throughout. There is little difficulty in fitting the book into a one-semester, or two-quarter course in linear network theory and design. It can be used equally well as a text in advanced network synthesis. For example, as an advanced text in modern network synthesis, Chapters 2, 4 and 5 plus some sections of Chapter 3 would serve for this purpose. Some of the later chapters are also suitable as topics for advanced seminars.

A special feature of the book is that results of direct practical value are included. They are design curves and tables for networks having Butterworth, Chebyshev or elliptic response. These results

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