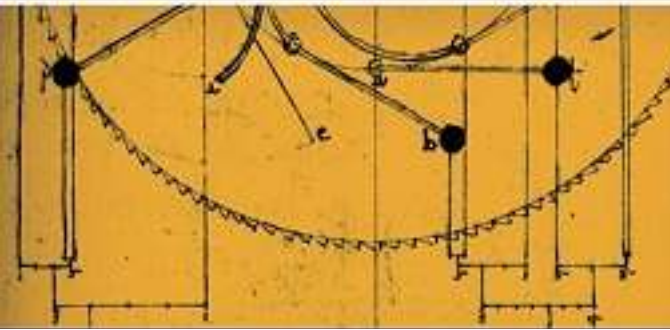


THIRD EDITION



ENGINEERING MECHANICS

# STATICS



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# *Engineering Mechanics*



## **STATICS**

Third Edition

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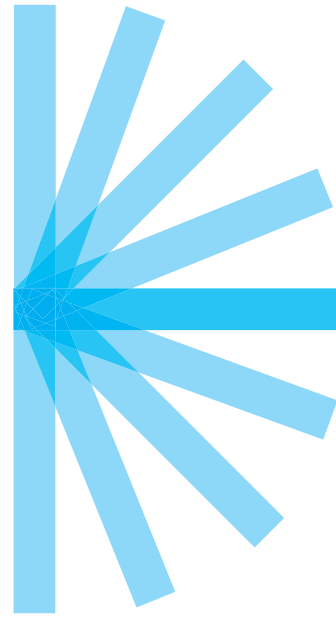
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***Engineering Mechanics***

***Statics***

**Third Edition**



***Andrew Pytel***  
***The Pennsylvania State University***

***Jaan Kiusalaas***  
***The Pennsylvania State University***



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Australia · Brazil · Japan · Korea · Mexico · Singapore · Spain · United Kingdom · United States

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Andrew Pytel and Jaan Kiusalaas**

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*To Jean, Leslie, Lori, John, Nicholas*

*and*

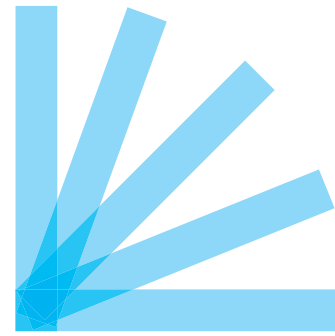
*To Judy, Nicholas, Jennifer, Timothy*

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\* Indicates optional articles

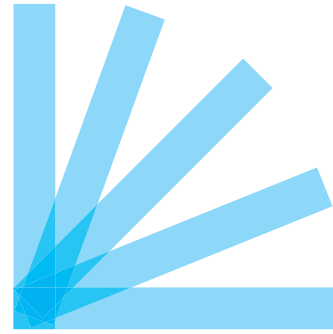
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# Preface



Statics and dynamics are basic subjects in the general field known as engineering mechanics. At the risk of oversimplifying, engineering mechanics is that branch of engineering that is concerned with the behavior of bodies under the action of forces. Statics and dynamics form the basis for many of the traditional fields of engineering, such as automotive engineering, civil engineering, and mechanical engineering. In addition, these subjects often play fundamental roles when the principles of mechanics are applied to such diverse fields as medicine and biology. Applying the principles of statics and dynamics to such a wide range of applications requires reasoning and practice rather than memorization. Although the principles of statics and dynamics are relatively few, they can only be truly mastered by studying and analyzing problems. Therefore, all modern textbooks, including ours, contain a large number of problems to be solved by the student. Learning the engineering approach to problem solving is one of the more valuable lessons to be learned from the study of statics and dynamics.

We have made every effort to improve our presentation without compromising the following principles that formed the basis of the previous editions.

- Each sample problem is carefully chosen to help students master the intricacies of engineering problem analysis.
- The selection of homework problems is balanced between “textbook” problems that illustrate the principles of engineering mechanics in a straight-forward manner, and practical engineering problems that are applicable to engineering design.
- The number of problems using U.S. Customary Units and SI Units are approximately equal.
- The importance of correctly drawn free-body diagrams is emphasized throughout.
- We continue to present equilibrium analysis in three separate articles, each followed by a set of problems. The first article teaches the method for drawing free-body diagrams. The second shows how to write and solve the equilibrium equations using a given free-body diagram. The third article combines the two techniques just learned to arrive at a logical plan for the complete analysis of an equilibrium problem.
- Whenever applicable, the number of independent equations is compared to the number of unknown quantities before the governing equations are written.
- Review Problems appear at the end of chapters to encourage students to synthesize the individual topics they have been learning.

We have included several optional topics, which are marked with an asterisk (\*). Due to time constraints, topics so indicated can be omitted without jeopardizing the presentation of the subject. An asterisk is also used to indicate problems that require advanced reasoning. Articles, sample problems, and problems associated with numerical methods are preceded by an icon representing a computer disk. 

In this third edition, we have made a number of significant improvements based upon the feedback received from students and faculty who have used the previous editions. In addition, we have incorporated many of the suggestions provided by the reviewers of the second edition.

A number of articles have been reorganized, or rewritten, to make the topics easier for the student to understand. For example, our presentation of beam analysis in Chapter 6 has been completely rewritten and includes both revised sample problems and revised problems. Our discussion of beams now more clearly focuses upon the methods and terminology used in the engineering analysis and design of beams. Also, the topic of rolling resistance has been added to Chapter 7. Furthermore, our discussion of virtual displacements in Chapter 10 has been made more concise and therefore will be easier for the students to understand. New to this edition, sections entitled Review of Equations have been added at the end of each chapter as a convenience for students as they solve the problems.

The total numbers of sample problems and problems remain about the same as in the previous edition; however, the introduction of two colors improves the overall readability of the text and artwork. Compared with the previous edition, approximately one-third of the problems is new, or has been modified.

**Ancillary** *Study Guide to Accompany Pytel and Kiusalaas Engineering Mechanics, Statics, Third Edition*, J.L. Pytel and A. Pytel, 2010. The goals of this study guide are two-fold. First, self-tests are included to help the student focus on the salient features of the assigned reading. Second, the study guide uses “guided” problems that give the student an opportunity to work through representative problems, before attempting to solve the problems in the text.

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ANDREW PYTEL  
JAAN KIUSALAAS

# 1

## Introduction to Statics



*The Flemish mathematician and engineer Simon Stevinus (1548–1620) was the first to demonstrate resolution of forces, thereby establishing the foundation of modern statics.*  
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### 1.1 Introduction

#### a. *What is engineering mechanics?*

Statics and dynamics are among the first engineering topics encountered by most students. Therefore, it is appropriate that we begin with a brief exposition on the meaning of the term *engineering mechanics* and on the role that these courses play in engineering education. Before defining engineering mechanics, we must first consider the similarities and differences between physics and engineering.

In general terms, *physics* is the science that relates the properties of matter and energy, excluding biological and chemical effects. Physics includes the study

of mechanics,\* thermodynamics, electricity and magnetism, and nuclear physics. On the other hand, *engineering* is the application of the mathematical and physical sciences (physics, chemistry, and biology) to the design and manufacture of items that benefit humanity. *Design* is the key concept that distinguishes engineers from scientists. According to the Accreditation Board for Engineering and Technology (ABET), engineering design is the process of devising a system, component, or process to meet desired needs.

*Mechanics* is the branch of physics that considers the action of forces on bodies or fluids that are *at rest* or *in motion*. Correspondingly, the primary topics of mechanics are statics and dynamics. The first topic that you studied in your initial physics course, in either high school or college, was undoubtedly mechanics. Thus, *engineering mechanics* is the branch of engineering that applies the principles of mechanics to mechanical design (i.e., any design that must take into account the effect of forces). The primary goal of engineering mechanics courses is to introduce the student to the engineering applications of mechanics. Statics and Dynamics are generally followed by one or more courses that introduce material properties and deformation, usually called Strength of Materials or Mechanics of Materials. This sequence of courses is then followed by formal training in mechanical design.

Of course, engineering mechanics is an integral component of the education of engineers whose disciplines are related to the mechanical sciences, such as aerospace engineering, architectural engineering, civil engineering, and mechanical engineering. However, a knowledge of engineering mechanics is also useful in most other engineering disciplines, because there, too, the mechanical behavior of a body or fluid must often be considered. Because mechanics was the first physical science to be applied to everyday life, it follows that engineering mechanics is the oldest branch of engineering. Given the interdisciplinary character of many engineering applications (e.g., robotics and manufacturing), a sound training in engineering mechanics continues to be one of the more important aspects of engineering education.

### **b.** *Problem formulation and the accuracy of solutions*

Your mastery of the principles of engineering mechanics will be reflected in your ability to formulate and solve problems. Unfortunately, there is no simple method for teaching problem-solving skills. Nearly all individuals require a considerable amount of practice in solving problems before they begin to develop the analytical skills that are so necessary for success in engineering. For this reason, a relatively large number of sample problems and homework problems are placed at strategic points throughout this text.

To help you develop an “engineering approach” to problem analysis, you will find it instructive to divide your solution for each homework problem into the following parts:

- 1. GIVEN:** After carefully reading the problem statement, list all the data provided. If a figure is required, sketch it neatly and approximately to scale.
- 2. FIND:** State precisely the information that is to be determined.

\*When discussing the topics included in physics, the term *mechanics* is used without a modifier. Quite naturally, this often leads to confusion between “mechanics” and “engineering mechanics.”

3. SOLUTION: Solve the problem, showing all the steps that you used in the analysis. Work neatly so that your work can be easily followed by others.
4. VALIDATE: Many times, an invalid solution can be uncovered by simply asking yourself, “Does the answer make sense?”

When reporting your answers, use only as many digits as the least accurate value in the given data. For example, suppose that you are required to convert 12 500 ft (assumed to be accurate to three significant digits) to miles. Using a calculator, you would divide 12 500 ft by 5280 ft/mi and report the answer as 2.37 mi (three significant digits), although the quotient displayed on the calculator would be 2.367 424 2. Reporting the answer as 2.367 424 2 implies that all eight digits are significant, which is, of course, untrue. It is your responsibility to round off the answer to the correct number of digits. *In this text*, you should assume that given data are accurate to three significant digits unless stated otherwise. For example, a length that is given as 3 ft should be interpreted as 3.00 ft.

When performing intermediate calculations, a good rule of thumb is to carry one more digit than will be reported in the final answer; for example, use four-digit intermediate values if the answer is to be significant to three digits. Furthermore, it is common practice to report four digits if the first digit in an answer is 1; for example, use 1.392 rather than 1.39.

## 1.2 *Newtonian Mechanics*

### a. *Scope of Newtonian mechanics*

In 1687 Sir Isaac Newton (1642–1727) published his celebrated laws of motion in *Principia (Mathematical Principles of Natural Philosophy)*. Without a doubt, this work ranks among the most influential scientific books ever published. We should not think, however, that its publication immediately established classical mechanics. Newton’s work on mechanics dealt primarily with celestial mechanics and was thus limited to particle motion. Another two hundred or so years elapsed before rigid-body dynamics, fluid mechanics, and the mechanics of deformable bodies were developed. Each of these areas required new axioms before it could assume a usable form.

Nevertheless, Newton’s work is the foundation of classical, or Newtonian, mechanics. His efforts have even influenced two other branches of mechanics, born at the beginning of the twentieth century: relativistic and quantum mechanics. *Relativistic mechanics* addresses phenomena that occur on a cosmic scale (velocities approaching the speed of light, strong gravitational fields, etc.). It removes two of the most objectionable postulates of Newtonian mechanics: the existence of a fixed or inertial reference frame and the assumption that time is an absolute variable, “running” at the same rate in all parts of the universe. (There is evidence that Newton himself was bothered by these two postulates.) *Quantum mechanics* is concerned with particles on the atomic or subatomic scale. It also removes two cherished concepts of classical mechanics: determinism and continuity. Quantum mechanics is essentially a probabilistic theory; instead of predicting an event, it determines the likelihood that an event will occur. Moreover, according to this theory, the events occur in discrete steps (called *quanta*) rather than in a continuous manner.



Relativistic and quantum mechanics, however, have by no means invalidated the principles of Newtonian mechanics. In the analysis of the motion of bodies encountered in our everyday experience, both theories converge on the equations of Newtonian mechanics. Thus the more esoteric theories actually reinforce the validity of Newton's laws of motion.

### b. *Newton's laws for particle motion*

Using modern terminology, Newton's laws of particle motion may be stated as follows:

1. If a particle is at rest (or moving with constant velocity in a straight line), it will remain at rest (or continue to move with constant velocity in a straight line) unless acted upon by a force.
2. A particle acted upon by a force will accelerate in the direction of the force. The magnitude of the acceleration is proportional to the magnitude of the force and inversely proportional to the mass of the particle.
3. For every action, there is an equal and opposite reaction; that is, the forces of interaction between two particles are equal in magnitude and oppositely directed along the same line of action.

Although the first law is simply a special case of the second law, it is customary to state the first law separately because of its importance to the subject of statics.

### c. *Inertial reference frames*

When applying Newton's second law, attention must be paid to the coordinate system in which the accelerations are measured. An *inertial reference frame* (also known as a Newtonian or Galilean reference frame) is defined to be any rigid coordinate system in which Newton's laws of particle motion relative to that frame are valid with an acceptable degree of accuracy. In most design applications used on the surface of the earth, an inertial frame can be approximated with sufficient accuracy by attaching the coordinate system to the earth. In the study of earth satellites, a coordinate system attached to the sun usually suffices. For interplanetary travel, it is necessary to use coordinate systems attached to the so-called fixed stars.

It can be shown that any frame that is translating with constant velocity relative to an inertial frame is itself an inertial frame. It is a common practice to omit the word *inertial* when referring to frames for which Newton's laws obviously apply.

### d. *Units and dimensions*

The standards of measurement are called *units*. The term *dimension* refers to the type of measurement, regardless of the units used. For example, kilogram and feet/second are units, whereas mass and length/time are dimensions. Throughout this text we use two standards of measurement: U.S. Customary system and SI system (from *Système internationale d'unités*). In the *U.S. Customary system* the base (fundamental) dimensions\* are force [ $F$ ], length [ $L$ ], and time [ $T$ ]. The corresponding base units are pound (lb), foot (ft), and second (s). The base dimensions in the *SI system* are mass [ $M$ ], length [ $L$ ], and time [ $T$ ], and the base units

\*We follow the established custom and enclose dimensions in brackets.

are kilogram (kg), meter (m), and second (s). All other dimensions or units are combinations of the base quantities. For example, the dimension of velocity is  $[L/T]$ , the units being ft/s, m/s, and so on.

A system with the base dimensions  $[FLT]$  (such as the U.S. Customary system) is called a *gravitational system*. If the base dimensions are  $[MLT]$  (as in the SI system), the system is known as an *absolute system*. In each system of measurement, the base units are defined by physically reproducible phenomena or physical objects. For example, the second is defined by the duration of a specified number of radiation cycles in a certain isotope, the kilogram is defined as the mass of a certain block of metal kept near Paris, France, and so on.

All equations representing physical phenomena must be *dimensionally homogeneous*; that is, each term of an equation must have the same dimension. Otherwise, the equation will not make physical sense (it would be meaningless, for example, to add a force to a length). Checking equations for dimensional homogeneity is a good habit to learn, as it can reveal mistakes made during algebraic manipulations.

### e. Mass, force, and weight

If a force  $\mathbf{F}$  acts on a particle of mass  $m$ , Newton's second law states that

$$\mathbf{F} = m\mathbf{a} \quad (1.1)$$

where  $\mathbf{a}$  is the acceleration vector of the particle. For a gravitational  $[FLT]$  system, dimensional homogeneity of Eq. (1.1) requires the dimension of mass to be

$$[M] = \left[ \frac{FT^2}{L} \right] \quad (1.2a)$$

In the U.S. Customary system, the derived unit of mass is called a *slug*. A slug is defined as the mass that is accelerated at the rate of  $1.0 \text{ ft/s}^2$  by a force of  $1.0 \text{ lb}$ . Substituting units for dimensions in Eq. (1.2a), we get for the unit of a slug

$$1.0 \text{ slug} = 1.0 \text{ lb} \cdot \text{s}^2/\text{ft}$$

For an absolute  $[MLT]$  system of units, dimensional homogeneity of Eq. (1.1) yields for the dimension of force

$$[F] = \left[ \frac{ML}{T^2} \right] \quad (1.2b)$$

The derived unit of force in the SI system is a *newton* (N), defined as the force that accelerates a  $1.0\text{-kg}$  mass at the rate of  $1.0 \text{ m/s}^2$ . From Eq. (1.2b), we obtain

$$1.0 \text{ N} = 1.0 \text{ kg} \cdot \text{m/s}^2$$

*Weight* is the force of gravitation acting on a body. Denoting gravitational acceleration (free-fall acceleration of the body) by  $g$ , the weight  $W$  of a body of mass  $m$  is given by Newton's second law as

$$W = mg \quad (1.3)$$

Note that mass is a constant property of a body, whereas weight is a variable that depends on the local value of  $g$ . The gravitational acceleration on the surface of the earth is approximately  $32.2 \text{ ft/s}^2$ , or  $9.81 \text{ m/s}^2$ . Thus the mass of a body that weighs  $1.0 \text{ lb}$  on earth is  $(1.0 \text{ lb})/(32.2 \text{ ft/s}^2) = 1/32.2 \text{ slug}$ . Similarly, if the mass of a body is  $1.0 \text{ kg}$ , its weight on earth is  $(9.81 \text{ m/s}^2)(1.0 \text{ kg}) = 9.81 \text{ N}$ .

At one time, the pound was also used as a unit of mass. The *pound mass* (lbm) was defined as the mass of a body that weighs  $1.0 \text{ lb}$  on the surface of the earth. Although pound mass is an obsolete unit, it is still used occasionally, giving rise to confusion between mass and weight. In this text, we use the pound exclusively as a unit of force.

### f. Conversion of units

A convenient method for converting a measurement from one set of units to another is to multiply the measurement by appropriate conversion factors. For example, to convert  $240 \text{ mi/h}$  into  $\text{ft/s}$ , we proceed as follows:

$$240 \text{ mi/h} = 240 \frac{\cancel{\text{mi}}}{\cancel{\text{h}}} \times \frac{1.0 \cancel{\text{h}}}{3600 \text{ s}} \times \frac{5280 \text{ ft}}{1.0 \cancel{\text{mi}}} = 352 \text{ ft/s}$$

where the multipliers  $1.0 \text{ h}/3600 \text{ s}$  and  $5280 \text{ ft}/1.0 \text{ mi}$  are conversion factors. Because  $1.0 \text{ h} = 3600 \text{ s}$  and  $5280 \text{ ft} = 1.0 \text{ mi}$ , we see that each conversion factor is dimensionless and of magnitude 1. Therefore, a measurement is unchanged when it is multiplied by conversion factors—only its units are altered. Note that it is permissible to cancel units during the conversion as if they were algebraic quantities.

Conversion factors applicable to mechanics are listed inside the front cover of the book.

### g. Law of gravitation

In addition to his many other accomplishments, Newton also proposed the law of universal gravitation. Consider two particles of mass  $m_A$  and  $m_B$  that are separated by a distance  $R$ , as shown in Fig. 1.1. The law of gravitation states that the two particles are attracted to each other by forces of magnitude  $F$  that act along the line connecting the particles, where

$$F = G \frac{m_A m_B}{R^2} \quad (1.4)$$

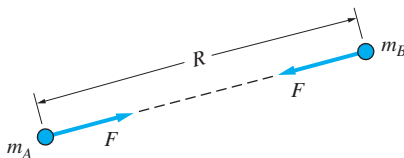


Fig. 1.1

The universal gravitational constant  $G$  is equal to  $3.44 \times 10^{-8} \text{ ft}^4/(\text{lb} \cdot \text{s}^4)$ , or  $6.67 \times 10^{-11} \text{ m}^3/(\text{kg} \cdot \text{s}^2)$ . Although this law is valid for particles, Newton showed that it is also applicable to spherical bodies, provided that their masses are distributed uniformly. (When attempting to derive this result, Newton was forced to develop calculus.)

If we let  $m_A = M_e$  (the mass of the earth),  $m_B = m$  (the mass of a body), and  $R = R_e$  (the mean radius of the earth), then  $F$  in Eq. (1.4) will be the weight  $W$  of the body. Comparing  $W = GM_e m/R_e^2$  with  $W = mg$ , we find that  $g = GM_e/R_e^2$ . Of course, adjustments may be necessary in the value of  $g$  for some applications in order to account for local variation of the gravitational attraction.

## Sample Problem 1.1

Convert 5000 lb/in.<sup>2</sup> to Pa (1 Pa = 1 N/m<sup>2</sup>).

### Solution

Using the conversion factors listed inside the front cover, we obtain

$$\begin{aligned} 5000 \text{ lb/in.}^2 &= 5000 \frac{\text{lb}}{\text{in.}^2} \times \frac{4.448 \text{ N}}{1.0 \text{ lb}} \times \left( \frac{39.37 \text{ in.}}{1.0 \text{ m}} \right)^2 \\ &= 34.5 \times 10^6 \text{ N/m}^2 = 34.5 \text{ MPa} \end{aligned} \quad \text{Answer}$$

## Sample Problem 1.2

The acceleration  $a$  of a particle is related to its velocity  $v$ , its position coordinate  $x$ , and time  $t$  by the equation

$$a = Ax^3t + Bvt^2 \quad (\text{a})$$

where  $A$  and  $B$  are constants. The dimension of the acceleration is length per unit time squared; that is,  $[a] = [L/T^2]$ . The dimensions of the other variables are  $[v] = [L/T]$ ,  $[x] = [L]$ , and  $[t] = [T]$ . Derive the dimensions of  $A$  and  $B$  if Eq. (a) is to be dimensionally homogeneous.

### Solution

For Eq. (a) to be dimensionally homogeneous, the dimension of each term on the right-hand side of the equation must be  $[L/T^2]$ , the same as the dimension for  $a$ . Therefore, the dimension of the first term on the right-hand side of Eq. (a) becomes

$$[Ax^3t] = [A][x^3][t] = [A][L^3][T] = \left[ \frac{L}{T^2} \right] \quad (\text{b})$$

Solving Eq.(b) for the dimension of  $A$ , we find

$$[A] = \frac{1}{[L^3][T]} \left[ \frac{L}{T^2} \right] = \frac{1}{[L^2T^3]} \quad \text{Answer}$$

Performing a similar dimensional analysis on the second term on the right-hand side of Eq. (a) gives

$$[Bvt^2] = [B][v][t^2] = [B] \left[ \frac{L}{T} \right] [T^2] = \left[ \frac{L}{T^2} \right] \quad (c)$$

Solving Eq. (c) for the dimension of  $B$ , we find

$$[B] = \left[ \frac{L}{T^2} \right] \left[ \frac{T}{L} \right] \left[ \frac{1}{T^2} \right] = \left[ \frac{1}{T^3} \right] \quad \text{Answer}$$

### Sample Problem 1.3

Find the gravitational force exerted by the earth on a 70-kg man whose elevation above the surface of the earth equals the radius of the earth. The mass and radius of the earth are  $M_e = 5.9742 \times 10^{24}$  kg and  $R_e = 6378$  km, respectively.

#### Solution

Consider a body of mass  $m$  located at the distance  $2R_e$  from the center of the earth (of mass  $M_e$ ). The law of universal gravitation, from Eq. (11.4), states that the body is attracted to the earth by the force  $F$  given by

$$F = G \frac{mM_e}{(2R_e)^2}$$

where  $G = 6.67 \times 10^{-11}$  m<sup>3</sup>/(kg · s<sup>2</sup>) is the universal gravitational constant. Substituting the values for  $G$  and the given parameters, the earth's gravitational force acting on the 70-kg man is

$$F = (6.67 \times 10^{-11}) \frac{(70)(5.9742 \times 10^{24})}{[2(6378 \times 10^3)]^2} = 171.4 \text{ N} \quad \text{Answer}$$

## Problems

- 1.1** A person weighs 30 lb on the moon, where  $g = 5.32 \text{ ft/s}^2$ . Determine (a) the mass of the person and (b) the weight of the person on earth.
- 1.2** The radius and length of a steel cylinder are 60 mm and 120 mm, respectively. If the mass density of steel is  $7850 \text{ kg/m}^3$ , determine the weight of the cylinder in pounds.
- 1.3** Convert the following: (a)  $400 \text{ lb} \cdot \text{ft}$  to  $\text{kN} \cdot \text{m}$ ; (b)  $6 \text{ m/s}$  to  $\text{mi/h}$ ; (c)  $20 \text{ lb/in.}^2$  to  $\text{kPa}$ ; and (d)  $500 \text{ slug/in.}$  to  $\text{kg/m}$ .
- 1.4** The mass moment of inertia of a certain body is  $I = 20 \text{ kg} \cdot \text{m}^2$ . Express  $I$  in terms of the base units of the U.S. Customary system.
- 1.5** The kinetic energy of a car of mass  $m$  moving with velocity  $v$  is  $E = mv^2/2$ . If  $m = 1000 \text{ kg}$  and  $v = 6 \text{ m/s}$ , compute  $E$  in (a)  $\text{kN} \cdot \text{m}$ ; and (b)  $\text{lb} \cdot \text{ft}$ .
- 1.6** In a certain application, the acceleration  $a$  and the position coordinate  $x$  of a particle are related by

$$a = \frac{gkx}{W}$$

where  $g$  is the gravitational acceleration,  $k$  is a constant, and  $W$  is the weight of the particle. Show that this equation is dimensionally consistent if the dimension of  $k$  is  $[F/L]$ .

- 1.7** When a force  $F$  acts on a linear spring, the elongation  $x$  of the spring is given by  $F = kx$ , where  $k$  is called the stiffness of the spring. Determine the dimension of  $k$  in terms of the base dimensions of an absolute  $[MLT]$  system of units.
- 1.8** In some applications dealing with very high speeds, the velocity is measured in  $\text{mm}/\mu\text{s}$ . Convert  $25 \text{ mm}/\mu\text{s}$  into (a)  $\text{m/s}$ ; and (b)  $\text{mi/h}$ .
- 1.9** A geometry textbook gives the equation of a parabola as  $y = x^2$ , where  $x$  and  $y$  are measured in inches. How can this equation be dimensionally correct?
- 1.10** The mass moment of inertia  $I$  of a homogeneous sphere about its diameter is  $I = (2/5)mR^2$ , where  $m$  and  $R$  are its mass and radius, respectively. Find the dimension of  $I$  in terms of the base dimensions of (a) a gravitational  $[FLT]$  system and (b) an absolute  $[MLT]$  system.
- 1.11** The position coordinate  $x$  of a particle is determined by its velocity  $v$  and the elapsed time  $t$  as follows: (a)  $x = At^2 - Bvt$ ; and (b)  $x = Avte^{-Bt}$ . Determine the dimensions of constants  $A$  and  $B$  in each case, assuming the expressions to be dimensionally correct.

**\*1.12** In a certain vibration problem the differential equation describing the motion of a particle of mass  $m$  is

$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = P_0 \sin \omega t$$

where  $x$  is the displacement of the particle and  $t$  is time. What are the dimensions of the constants  $c$ ,  $k$ ,  $P_0$ , and  $\omega$  in terms of the base dimensions of a gravitational  $[FLT]$  system?

**1.13** Using Eq. (1.4), derive the dimensions of the universal gravitational constant  $G$  in terms of the base dimensions of (a) a gravitational  $[FLT]$  system; and (b) an absolute  $[MLT]$  system.

**1.14** The typical power output of a compact car engine is 120 hp. What is the equivalent power in (a)  $\text{lb} \cdot \text{ft/s}$ ; and (b) kW?

**1.15** Two 10-kg spheres are placed 500 mm apart. Express the gravitational attraction acting on one of the spheres as a percentage of its weight on earth.

**1.16** Two identical spheres of radius 8 in. and weighing 2 lb on the surface of the earth are placed in contact. Find the gravitational attraction between them.

Use the following data for Problems 1.17–1.21: mass of earth =  $5.9742 \times 10^{24}$  kg, radius of earth = 6378 km, mass of moon =  $0.073483 \times 10^{24}$  kg, radius of moon = 1737 km.

**1.17** A man weighs 180 lb on the surface of the earth. Compute his weight in an airplane flying at an elevation of 30 000 ft.

**1.18** Use Eq. (1.4) to show that the weight of an object on the moon is approximately 1/6 its weight on earth.

**1.19** Plot the earth's gravitational acceleration  $g$  ( $\text{m/s}^2$ ) against the height  $h$  (km) above the surface of the earth.

**1.20** Find the elevation  $h$  (km) where the weight of an object is one-tenth its weight on the surface of the earth.

**1.21** Calculate the gravitational force between the earth and the moon in newtons. The distance between the earth and the moon is  $384 \times 10^3$  km.

### 1.3 Fundamental Properties of Vectors

A knowledge of vectors is a prerequisite for the study of statics. In this article, we describe the fundamental properties of vectors, with subsequent articles discussing some of the more important elements of vector algebra. (The calculus of vectors will be introduced as needed in *Dynamics*.) We assume that you are already familiar with vector algebra—our discussion is intended only to be a review of the basic concepts.

The differences between scalar and vector quantities must be understood:

A *scalar* is a quantity that has magnitude only. A *vector* is a quantity that possesses magnitude and direction and obeys the parallelogram law for addition.

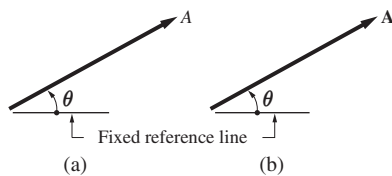
Because scalars possess only magnitudes, they are real numbers that can be positive, negative, or zero. Physical quantities that are scalars include temperature, time, and speed. As shown later, force, velocity, and displacement are examples of physical quantities that are vectors. The magnitude of a vector is always taken to be a nonnegative number. When a vector represents a physical quantity, the units of the vector are taken to be the same as the units of its magnitude (pounds, meters per second, feet, etc.).

The algebraic notation used for a scalar quantity must, of course, be different from that used for a vector quantity. In this text, we adopt the following conventions: (1) scalars are written as italicized English or Greek letters—for example,  $t$  for time and  $\theta$  for angle; (2) vectors are written as boldface letters—for example,  $\mathbf{F}$  for force; and (3) the magnitude of a vector  $\mathbf{A}$  is denoted as  $|\mathbf{A}|$  or simply as  $A$  (italic).

There is no universal method for indicating vector quantities when writing by hand. The more common notations are  $\vec{A}$ ,  $\underline{A}$ ,  $\bar{A}$ , and  $\underline{A}$ . Unless instructed otherwise, you are free to use the convention that you find most comfortable. However, it is imperative that you take care to always distinguish between scalars and vectors when you write.

The following summarizes several important properties of vectors.

**Vectors as Directed Line Segments** Any vector  $\mathbf{A}$  can be represented geometrically as a directed line segment (an arrow), as shown in Fig. 1.2(a). The magnitude of  $\mathbf{A}$  is denoted by  $A$ , and the direction of  $\mathbf{A}$  is specified by the sense of the arrow and the angle  $\theta$  that it makes with a fixed reference line. When using graphical methods, the length of the arrow is drawn proportional to the magnitude of the vector. Observe that the representation shown in Fig. 1.2(a) is complete because both the magnitude and direction of the vector are indicated. In some instances, it is also convenient to use the representation shown in Fig. 1.2(b), where the vector character of  $\mathbf{A}$  is given additional emphasis by using boldface. Both of these representations for vectors are used in this text.



**Fig. 1.2**

We see that a vector does not possess a unique line of action, because moving a vector to a parallel line of action changes neither its magnitude nor its direction. In some engineering applications, the definition of a vector is more restrictive to include a line of action or even a point of application—see Art. 2.2.



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