

Nature's Patterns: a tapestry in three parts



Flow

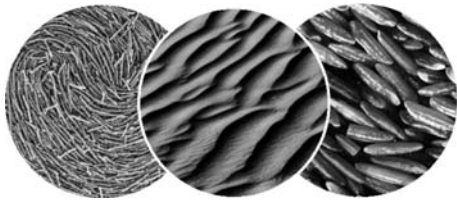
PHILIP BALL

Nature's Patterns

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Nature's Patterns

A Tapestry in Three Parts



Philip Ball

Nature's Patterns is a trilogy composed of
Shapes, Flow, and Branches

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Flow



MOVEMENT creates pattern and form. Moving water arranges itself into eddies, and sometimes places these in strict array, where they become baroque and orderly conduits for unceasing flow. The motions of air and water organize the skies, the earth, and the oceans. The hidden logic of gases in turmoil paints great spinning eyes on the outer planets. Out of the collisions of particles in motion, desert dunes arise and hills become striped with sorted grains. Give these grains the ability to respond to their neighbours—make them fish, or birds, or buffalos—and there seems no end to the patterns that may appear, each an extraordinary collaboration that no individual has ordained or planned.

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Preface and acknowledgements



AFTER my 1999 book *The Self-Made Tapestry: Pattern Formation in Nature* went out of print, I'd often be contacted by would-be readers asking where they could get hold of a copy. That was how I discovered that copies were changing hands in the used-book market for considerably more than the original cover price. While that was gratifying in its way, I would far rather see the material accessible to anyone who wanted it. So I approached Latha Menon at Oxford University Press to ask about a reprinting. But Latha had something more substantial in mind, and that is how this new trilogy came into being. Quite rightly, Latha perceived that the original *Tapestry* was neither conceived nor packaged to the best advantage of the material. I hope this format does it more justice.

The suggestion of partitioning the material between three volumes sounded challenging at first, but once I saw how it might be done, I realized that this offered a structure that could bring more thematic organization to the topic. Each volume is self-contained and does not depend on one having read the others, although there is inevitably some cross-referencing. Anyone who has seen *The Self-Made Tapestry* will find some familiar things here, but also plenty that is new. In adding that material, I have benefited from the great generosity of many scientists who have given images, reprints and suggestions. I am particularly grateful to Sean Carroll, Iain Couzin, and Andrea Rinaldo for critical readings of some of the new text. Latha set me more work than I'd perhaps anticipated, but I remain deeply indebted to her for her vision of what these books might become, and her encouragement in making that happen.

Philip Ball

London, October 2007

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The Man Who Loved Fluids

Leonardo's Legacy



PERHAPS it is not so strange after all that the man who has come to personify polyvalent virtuosity, defining the concept of the Renaissance man and becoming a symbol for the unity of all learning and creative endeavour, was something of an under-achiever. That might seem an odd label to attach to Leonardo da Vinci, but the fact is that he started very little and finished even less. His life was a succession of plans made and never realized, of commissions refused (or accepted and never honoured), of studies undertaken with such a mixture of obsessive diligence and lack of system or objective that they could offer little instruction to future generations. This was not because Leonardo was a laggard; on the contrary, his ambitions often exceeded his capacity to fulfil them.

Yet if Leonardo did not achieve as much as we feel he might have done, that did not prevent his contemporaries from recognizing his extraordinary genius. The Italian artist and writer Giorgio Vasari was prone to eulogize all his subjects in his sixteenth-century *Lives of the Artists*, but he seems to make a special effort for Leonardo:

In the normal course of events many men and women are born with various remarkable qualities and talents; but occasionally, in a way that transcends nature, a single person is marvellously endowed by heaven with beauty, grace, and talent in such abundance that he leaves other men far behind, all his actions seem inspired, and indeed everything he does clearly comes from God rather than from human art. Everyone acknowledged that this was true of Leonardo da Vinci, an artist of outstanding physical beauty who displayed infinite grace in everything he did and who cultivated his genius so brilliantly that all problems he studied he solved with ease.

What Vasari did not wish to admit is that such an embarrassment of riches can be a burden rather than a blessing, and that it sometimes takes duller men to see a project through to its end while geniuses can only initiate them without cease. Leonardo's devotion to the study of nature and science could leave his artistic patrons frustrated. Isabella d'Este, marchesa of Mantua, was told by an emissary whom she dispatched to Florence to commission a portrait from the great painter, that 'he is working hard at geometry and is very impatient of painting . . . In short his mathematical experiments have so estranged him from painting that he cannot bear to take up a brush.'

But Leonardo was apt when the mood was upon him to labour without stint. His contemporary Matteo Bandello, a Piedmontese novelist, saw him at work on his ill-fated *Last Supper*: 'It was his habit often, and I have frequently seen him, to go early in the morning and mount upon the scaffolding . . . it was his habit, I say, from sunrise until dusk never to lay down his brush, but, forgetful alike of eating and drinking, to paint without intermission.' And yet his genius demanded space for reflection that he could ill afford. 'At other times', Bandello avers, 'two, three or four days would pass without his touching the fresco, but he would remain before it for an hour or two at a time merely looking at it, considering, examining the figures.' 'Oh dear, this man will never do anything!', Pope Leo X is said to have complained.

As his sketchbooks attest, lengthy and contemplative examination was his forte. When Leonardo looked at something, he saw more than other people. This was no idle gaze but an attempt to discern the very soul of things, the deep and elusive forms of nature. In his studies of anatomy, of animals and drapery, of plants and landscapes, and of ripples and torrents of water, he shows us things that transcend the naturalistic: shapes that we might not directly perceive ourselves but that we suspect we would if we had Leonardo's eyes.

We are accustomed to list Leonardo's talents as though trying to assign him to a university department: painter, sculptor, musician, anatomist, military and civil engineer, inventor, physicist. But his notebooks mock such distinctions. Rather, it seems that Leonardo was assailed by questions everywhere he looked, which he had hardly the opportunity or inclination to arrange into a systematic course of study. Is the sound of a blacksmith's labours made with in the hammer or the anvil? Which will fire farthest, gunpowder doubled in quantity or in quality? What is the shape of corn tossed in a sieve? Are the tides caused by the

Moon or the Sun, or by the 'breathing of the Earth'? From where do tears come, the heart or the brain? Why does a mirror exchange right and left? Leonardo scribbles these memos to himself in his cryptic left-handed script; sometimes he finds answers, but often the question is left hanging. On his 'to do' list are items that boggle the mind with their casual boldness: 'Make glasses in order to see the moon large.' It is no wonder that Leonardo had no students and founded no school, for his was an intensely personal enquiry into nature, one intended to satisfy no one's curiosity but his own.

We come no closer to understanding this quest, however, if we persist in seeing Leonardo as an artist on the one hand and a scientist and technologist on the other. The common response is to suggest that he recognized no divisions between the two, and he is regularly invoked to advertise the notion that both are complementary means of studying and engaging with nature. This doesn't quite hit the mark, however, because it tacitly accepts that 'art' and 'science' had the same connotations in Leonardo's day as they do now. What Leonardo considered *arte* was the business of making things. Paintings were made by *arte*, but so were the apothecaries' drugs and the weavers' cloth. Until the Renaissance there was nothing particularly admirable about art, or at least about artists—patrons admired fine pictures, but the people who made them were tradesmen paid to do a job, and manual workers at that. Leonardo himself strove to raise the status of painting so that it might rank among the 'intellectual' or liberal arts, such as geometry, music, and astronomy. Although a formidable sculptor himself, he argued his case by dismissing it as 'less intellectual': it is more enduring, admittedly, 'but excels in nothing else'. The academic and geometric character of treatises on painting at that time, most notably that of the polymath Leon Battista Alberti, which can make painting seem less a matter of inspiration than a process of drawing lines and plotting light rays, derives partly from this agenda.

Scienza, in contrast, was knowledge—but not necessarily that obtained by careful experiment and enquiry. Medieval scholastics had insisted that knowledge was what appeared in the books of Euclid, Aristotle, Ptolemy, and other ancient writers, and that the learned man was one who had memorized these texts. The celebrated humanism of the Renaissance did not challenge this idea but merely refreshed it, insisting on returning to the original sources rather than relying on Arabic and medieval glosses. In this regard, Leonardo was not a 'scientist', since he

was not well schooled—the humble son of a minor notary and a peasant woman, he was defensive all his life about his poor Latin and ignorance of Greek. He believed in the importance of *scienza*, certainly, but for him this did not consist solely of book-learning. It was an *active* pursuit, and demanded experiments, though Leonardo did not exactly conduct them in the manner that a modern scientist would. For him, true insight came from peering beneath the surface of things. That is why his painstaking studies of nature, while appearing superficially Aristotelian in their attention to particulars, actually have much more of a Platonic spirit: they are an attempt to see what is really there, not what appears to be. This is why he had to sit and stare for hours: not to see things more sharply, but, as it were, to *stop* seeing, to transcend the limitations of his eyes.

Leonardo regarded the task of the painter to be not naturalistic mimicry, which shows only the surface contours and shallow glimmers of the world, but the use of reason to shape his vision and distil from it a kind of universal truth. 'At this point', Leonardo wrote of those who would grow tired of his theoretical musings on the artist's task, 'the opponent says that he does not want so much *scienza*, that practice is enough for him in order to draw the things in nature. The answer to this is that there is nothing that deceives us more easily than our confidence in our judgement, divorced from reasoning.' This could have been written by Plato himself, famously distrustful of the deceptions of painters.

I hope you can start to appreciate why I have placed Leonardo centre stage in introducing this volume of my survey of nature's patterns. As I explained in Book I, the desire to look *through* nature and find its underlying forms and structures is what characterizes the approach of some of the pioneers in the study of pattern formation, such as the German biologist Ernst Haeckel and the Scottish zoologist D'Arcy Wentworth Thompson. Haeckel was another gifted artist who firmly believed that the natural world needs to be arranged, ordered, tidied, before its forms and generative impulses can be properly perceived. Thompson shared Leonardo's conviction that the similarities of form and pattern we see in very different situations—for Leonardo it might be the cascades of a water spout and a woman's hair—reveal a deep-seated relationship. D'Arcy Thompson's view of such correspondences is one we can still accept in science today, based as it is on the idea that the same forces are likely to be at play in both cases. Leonardo's rationalization is more

remote now from our experience, being rooted in the tradition of Neoplatonism that saw these correspondences as a central feature of nature's divine architecture: *as above, so below*, as the reductive formulation has it. When Leonardo calls rivers the blood of the Earth, and comments on how their channels resemble the veins of the human body, he is not engaging in some vague metaphor or visual pun; the two are related because the Earth is indeed a kind of living body and can therefore be expected to echo the structures of our own anatomy.

In this vision of a kind of hidden essence of nature, we can find the true nexus of Leonardo's 'art' and 'science'. We tend to think of his art as 'lifelike', and Vasari made the same mistake. He praises the vase of flowers that appears in one of Leonardo's Madonnas for its 'wonderful realism', but then goes on, I think inadvertently, to make a telling remark by saying that the flowers 'had on them dewdrops that looked more convincing than the real thing'. Leonardo might have answered that this was because he had indeed painted 'the real thing' and not what his eyes had shown him. His work is not photographic but stylized, synthetic, even abstract, and he admits openly that painting is a work not of imitation but of invention: 'a subtle *inventione* which with philosophy and subtle speculation considers the natures of all forms'. Leonardo 'is thinking of art not simply in technical terms', says art historian Adrian Parr, 'where the artist skillfully renders a form on the canvas... Rather, he takes the relationship of nature to art onto a deeper level, intending to express in his art "every kind of form produced in nature".' For indeed, as the art historian Martin Kemp explains, 'Leonardo saw nature as weaving an infinite variety of elusive patterns on the basic warp and woof of mathematical perfection.' And so, without a doubt, did D'Arcy Thompson.

Leonardian flows

While most painters used technique to create a simulacrum of nature, Leonardo felt that one could not imbue the picture with life until one understood how nature does it. His sketches, then, are not exactly studies but something between an experiment and a diagram—attempts to intuit the forces at play (Fig. 1.1). 'Leonardo's use of swirling, curving, revolving and wavy patterns, becomes a means of both investigating and entering into the rhythmic movements of nature', says Parr.

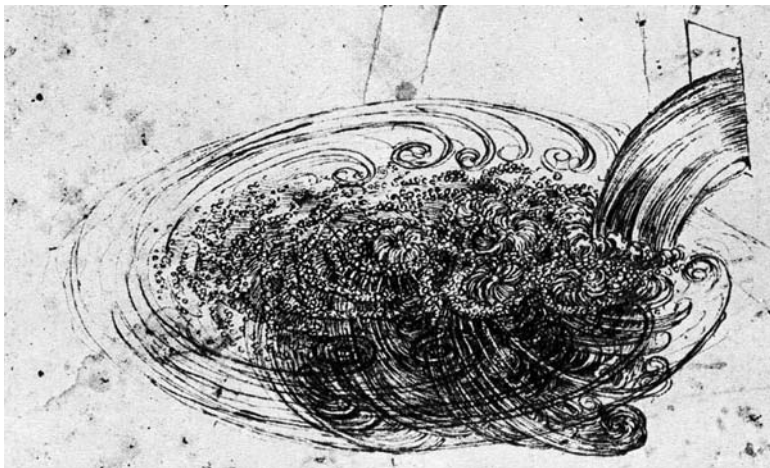


Fig. 1.1: A sketch of flowing water by Leonardo da Vinci.

Other western artists have tried to capture the forms of movement and flow, whether in the boiling vapours painted by J. M. W. Turner, the stop-frame dynamism of Marcel Duchamp's *Nude Descending a Staircase* (1912) or the fragmented frenzy of the Italian Futurists. But these are impressionistic, ad hoc and subjective efforts that lack Leonardo's scientific sense of pattern and order. Perhaps it is impossible truly to depict the world in this way unless you are a Neoplatonist. When John Constable declared in the early nineteenth century that 'Painting is a science and should be pursued as an inquiry into the laws of nature', he had in mind something far more mechanistic: that the painter should understand how physics and meteorology create a play of light and shadow, so that the paintings become convincing in an illusionistic sense.

But while a Leonardian perspective is valuable for surveying all nature's patterns, I have made him the pinion for this volume on patterns of motion, in fluids particularly, because there were few topics that enthralled him (and I mean that in its original sense) more than this. Of all the passions that he evinced, none seems more ardent than the wish to understand water. One senses that he regards it as the central elemental force: 'water is the driver of Nature', he says, 'It is never at rest until it unites with the sea... It is the expansion and humour of all vital bodies. Without it nothing retains its form.' It is no wonder, then, that one of Leonardo's most revealing and famous notebooks, known as the Codex

Leicester or Codex Hammer,* is mostly concerned with water. There was hardly an aspect of water that Leonardo did not leave unexamined. He wrote about sedimentation and erosion in rivers, and how they produce meanders and sand ripples on the river bed (two patterns I consider later). He discussed how water circulates on the Earth in what we now call the hydrological cycle, evaporating from the seas and falling as rain on to high ground. He asked why the sea is salty and wondered why a man can remain underwater only 'for such a time as he can hold his breath'. He investigated Archimedes spirals for lifting water, as well as suction pumps and water wheels. He drew astonishing 'aerial' pictures of river networks (we'll see them in Book III), and planned great works of hydraulic engineering. In collaboration with Niccolò Machiavelli, he drew up a scheme to redirect the flow of the Arno River away from Pisa, thereby depriving the city of its water supply and delivering it into the hands of the Florentines.

It seems that Leonardo did not become fascinated by water because of his engineering activities; rather, according to art historian Arthur Popham, the latter were a symptom of the former: 'Something in the movement of water, its swirls and eddies, corresponded to some deep-seated twist in his nature.' No aspect of water captured his interest more than the eddies of a flowing stream. He wrote long lists of the features of these vortices that he intended at some point to investigate:

Of eddies wide at the mouth and narrow at the base.
 Of eddies very wide at the base and narrow above.
 Of eddies of the shape of a column.
 Of eddies formed between two masses of water that rub together.

And so on—pages and pages of optimistic plans, of experiments half-started, of speculations and ideas, all described in such obsessive detail that even Leonardo scholars have pronounced them virtually unreadable. 'He wants', says the art historian Ernst Gombrich, 'to classify vortices as a zoologist classifies the species of animals.'

To judge from his sketches, Leonardo conducted a thorough, if haphazard, experimental programme on the flow patterns of water, watching it pass down channels of different shapes, charting the chaos of plunging waterfalls, and placing obstacles in the flow to see how they generated new forms. His drawings of water surging around the sides of a plate face-on to the flow show a delicately braided wake (Fig. 1.2*a*),

*The manuscript was acquired and published by Lord Leicester in Rome in the eighteenth century, but was bought in 1980 by the American Maecenas Armand Hammer.



Fig. 1.2: In the braided patterns of water flowing around a flat plate (a), Leonardo found echoes of the braids in a woman's hair (b).



Fig. 1.2: (Continued).

and the resemblance to the braided hair of a woman in a preparatory study (Fig. 1.2*b*) is no coincidence, for as Leonardo said himself,

Observe the motion of the surface of the water which resembles that of hair, which has two motions, of which one depends on the weight of the hair, the other on the direction of the curls; thus the water forms eddying whirlpools, one point of which is due to the impetus of the original current and the other to the incidental motion and return flow.

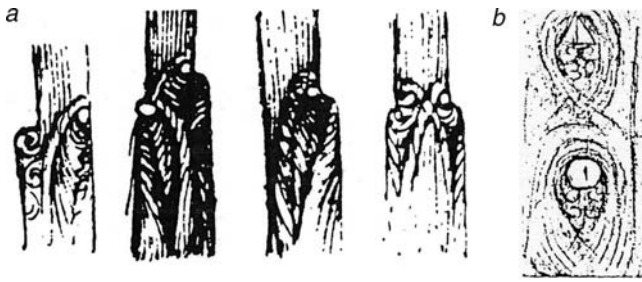


Fig. 1.3: Leonardo sketched shock waves caused by constrictions in a channel (a) and the shape of wakes in flow around an obstacle (b).

His self-portrait from 1512 shows his long hair and beard awash with eddies.

Many of these visual records are remarkably fine: he illustrates shock waves and ripples caused by constriction and widening of a channel (Fig. 1.3a), and his drawings of the flow past a cylindrical obstruction display the teardrop wake and the paired vortices (Fig. 1.3b) that have been found in modern experiments (see page 28). Fluid scientists today typically use techniques for revealing flow-forms that Leonardo is often said to have invented: fine particles that reflect light are suspended in the water, or coloured dyes are added to part of the flow. 'If you throw sawdust down into a running stream', Leonardo said,

you will be able to observe where the water turned upside down after striking against the banks throws this sawdust back towards the centre of the stream, and also the revolutions of the water and where other water either joins it or separates from it; and many other things.

Roughly speaking, these floating particles map out what are now called *streamlines*, which can be thought of as the trajectories of the flow.* In this sense, Leonardo's studies of flow patterns were thoroughly modern.

*A streamline has a technical definition: it is a line within the fluid for which the tangent at any point shows the direction of flow at that point. Streamlines show not only 'where the fluid is going' but also how fast: where streamlines are close together, the velocity is high. In steady flows, where the pattern of flow doesn't change over time, the path of a suspended particle or the trajectory of dye injected at a point, the particle path or so-called *streakline* of the dye trace out streamlines. But if the flow is unsteady, this is no longer true; the particle paths or streaklines can give an impression of the streamlines, and the true streamlines can be deduced from them, but they are not the same thing.

But he had only his eyes and his memory to guide him in translating from what he saw to what he drew; and as art historians know, that translation occurs in a context of preconceived notions of style and motif that condition what is depicted. When Leonardo compares a flow to hair, he is struck initially by the resemblance, but then this correspondence superimposes what he knows of the way hair falls on what he sees in the stream of water. The result is, as Popham says, that although

[t]he cinematographic vision which could see, the prodigious memory which could retain and the hand which could record these evanescent and intangible formations are little short of miraculous... [t]hese drawings do not so much convey the impression of water as of some exquisite submarine vegetable growth.

Was Leonardo able to do anything beyond recording what he perceived? Did he elucidate the reasons why these marvellous patterns are formed in water? If we have to admit that he did not really do that, it is no disgrace, since that problem is one of the hardest of all in physical science, and has still not been completely solved. On the whole the flows that Leonardo was studying were turbulent, fast-moving and unsteady in the extreme, so that they changed from one moment to the next. If he could describe these flows only in pictures and words, scientists could do no better than that until the twentieth century. And what vivid descriptions he gave!—

The whole mass of water, in its breadth, depth and height, is full of innumerable varieties of movements, as is shown on the surface of currents with a moderate degree of turbulence, in which one sees continually gurglings and eddies with various swirls formed by the more turbid water from the bottom as it rises to the surface.

Leonardo made some discoveries that stand up today. His comment that ‘water in straight rivers is swifter the farther it is from the shore, its impediment’, for example, is an elegant description of what fluid scientists call the velocity profile of flow in a channel, which is determined by the way friction between the fluid and the channel wall brings the flow there virtually to a standstill. Leonardo’s explanation of how river meanders are caused by shifting patterns of sedimentation and erosion by the flow contain all the elements that today’s earth scientists recognize.

His legacy for our understanding of fluid flow patterns goes deeper than this, however. As far as we can tell, Leonardo was the first Western scientist to really make the case that this phenomenon deserves serious study. And he showed that flowing water is not simply an unstructured

chaos but contains persistent forms that can be recognized, recorded, analysed—forms, moreover, that are things of great beauty, of value to the artist as well as the scientist.

Transcendental forms

All the same, Leonardo's idiosyncratic, hermetic way of working meant that no research programme stemmed from his achievements. No scientist seems subsequently to have thought very much about fluid flows until the Swiss mathematician Daniel Bernoulli began to investigate them in the seventeenth century.*

Nor did Leonardo's work on fluid motion have any artistic legacy: his studies of flows as a play of patterns, forms, and streamlines leave no trace in Western art. Artists looked instead for a stylized realism which insisted that turbulent water be depicted as a play of glinting highlights and surging foam: a style that is all surface, you might say. Just about any dramatic seascape of the eighteenth or nineteenth centuries will show this—George Morland's *The Wreckers* (1791) is a good example (Fig. 1.4).

A fluid style akin to Leonardo's does not show up again in Western art until the lively arabesques of the Art Nouveau movement of the late nineteenth century (Fig. 1.5). These artists took their inspiration from natural forms, such as the elegant curves and spirals of plant stems. As I discussed in Book I, the delicate frond-like forms discovered at this time in marine organisms and drawn with great panache and skill by Ernst Haeckel became a significant influence on the German branch of this movement, known as the Jugendstil—a two-way interaction that probably conditioned the way Haeckel drew in the first place. In England these trends produced something truly Leonardian in the works of the illustrator Arthur Rackham, where the correspondences between the waves and vortices of water, smoke, hair, and vegetation are particularly explicit (Fig. 1.6). But the use of vortical imagery here is really nothing more than a style, valued for its decorative and allusive qualities: there is no real sense that the artists are, like Leonardo,

*René Descartes made much of vortices, becoming convinced that the entire universe is filled with an ethereal fluid that swirls at all scales. Their gyrating motions, he said, carry along the heavenly bodies, explaining the circulations of the planets and stars. His theory, however, does not seem to owe any inspiration to Leonardo's work on eddies.



Fig. 1.4: *The Wreckers* by George Morland shows the typical manner in which Western painters depicted flow as a play of light. (Image: Copyright Southampton City Art Gallery, Hampshire, UK/ The Bridgeman Art Library.)

simultaneously conducting an investigation into nature's forms rather than simply adapting them for aesthetic ends.

One of the sources of the bold lines and sinuous forms of Art Nouveau is, however, more pertinent. In the mid-nineteenth century trade opened up between Western Europe and the Far East, and Japanese woodblock prints came into vogue among artist and collectors. Here Western artists found a very different way of depicting the world—not as naturalistic *chiaroscuro* but as a collage of flat, clearly delineated elements that disdains the rules of scientific optics and makes no pretence of photographic *trompe l'oeil*. To the Western eye these pictures are stylized and schematic, but some artists could see that this was not mere affectation, less still a simplification. What was being conveyed was the essence of things, unobstructed by superficial incidentals.

It is as simplistic to generalize about Chinese and Japanese art as it is about the art of the West—these traditions, too, have their different periods and schools and philosophies. But it is fair to say that most Chinese artists have attempted to imbue their works with *Ch'i*, the vital

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