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Bezalel Peleg
Peter Sudhölter

INTRODUCTION TO THE THEORY OF COOPERATIVE GAMES

SECOND EDITION

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Bezalel Peleg · Peter Sudhölter

Introduction to the Theory of Cooperative Games

Second Edition

 Springer

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Preface to the Second Edition

The main purpose of the second edition is to enhance and expand the treatment of games with nontransferable utility. The main changes are:

- (1) Chapter 13 is devoted entirely to the Shapley value and the Harsanyi solution. Section 13.4 is new and contains an axiomatization of the Harsanyi solution.
- (2) Chapter 14 deals exclusively with the consistent Shapley value. Sections 14.2 and 14.3 are new and present an existence proof for the consistent value and an axiomatization of the consistent value respectively. Section 14.1, which was part of the old Chapter 13, deals with the consistent value of polyhedral games.
- (3) Chapter 15 is almost entirely new. It is mainly devoted to an investigation of the Mas-Colell bargaining set of majority voting games. The existence of the Mas-Colell set is investigated and various limit theorems are proved for majority voting games. As a corollary of our results we show the existence of a four-person super-additive and non-levelled (NTU) game whose Mas-Colell bargaining set is empty.
- (4) The treatment of the ordinal bargaining set was moved to the final chapter 16.

We also have used this opportunity to remove typos and inaccuracies from Chapters 2 – 12 which otherwise remained intact.

We are indebted to all our readers who pointed out some typo. In particular we thank Michael Maschler for his comments and Martina Bihn who personally supported this edition.

June 2007

Bezalel Peleg and Peter Sudhölter

Preface to the First Edition

In this book we study systematically the main solutions of cooperative games: the core, bargaining set, kernel, nucleolus, and the Shapley value of TU games, and the core, the Shapley value, and the ordinal bargaining set of NTU games. To each solution we devote a separate chapter wherein we study its properties in full detail. Moreover, important variants are defined or even intensively analyzed. We also investigate in separate chapters continuity, dynamics, and geometric properties of solutions of TU games. Our study culminates in uniform and coherent axiomatizations of all the foregoing solutions (excluding the bargaining set).

It is our pleasure to acknowledge the help of the following persons and institutions. We express our gratitude to Michael Maschler for his detailed comments on an early version, due to the first author, of Chapters 2 – 8. We thank Michael Borns for the linguistic edition of the manuscript of this book. We are indebted to Claus-Jochen Haake, Sven Klauke, and Christian Weiß for reading large parts of the manuscript and suggesting many improvements. Peter Sudhölter is grateful to the Center for Rationality and Interactive Decision Theory of the Hebrew University of Jerusalem and to the Edmund Landau Center for Research in Mathematical Analysis and Related Areas, the Institute of Mathematics of the Hebrew University of Jerusalem, for their hospitality during the academic year 2000-01 and during the summer of 2002. These institutions made the typing of the manuscript possible. He is also grateful to the Institute of Mathematical Economics, University of Bielefeld, for its support during several visits in the years 2001 and 2002.

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Bezalel Peleg and Peter Sudhölter

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Notation and Symbols

We shall now list some of our notation.

The field of real numbers is denoted by \mathbb{R} and \mathbb{R}_+ is the set of nonnegative reals. For a finite set S , the Euclidean vector space of real functions with the domain S is denoted by \mathbb{R}^S . An element x of \mathbb{R}^S is represented by the vector $(x^i)_{i \in S}$. Also, $\mathbb{R}_+^S = \{x \in \mathbb{R}^S \mid x^i \geq 0 \text{ for all } i \in S\}$ and $\mathbb{R}_{++}^S = \{x \in \mathbb{R}^S \mid x^i > 0 \text{ for all } i \in S\}$. If $x, y \in \mathbb{R}^N$, $S, T \subseteq N$, and $S \cap T = \emptyset$, then $x^S = (x^i)_{i \in S}$ and $z = (x^S, y^T) \in \mathbb{R}^{S \cup T}$ is given by $z^i = x^i$ for all $i \in S$ and $z^j = y^j$ for all $j \in T$.

The symbols in the following list are ordered according to the page numbers, the numbers in the first column, of their definitions or first occurrences.

2	$X \setminus Y$	set difference ($\{x \in X \mid x \notin Y\}$)
9	\mathcal{U}	the universe of players
9	(N, v)	coalitional TU game
10	\Rightarrow	implies, implication
11	$x(S)$	aggregate amount of S
11	$\alpha v + \beta$	strategically equivalent coalition function to v
12	\mathcal{SYM}	group of symmetries
12	$ A $	cardinality of A
19	$X^*(N, v)$	feasible payoff vectors
19	Γ	set of TU games
19	σ	solution
19	$\mathcal{C}(N, v)$	the core
19	$\pi(x)$	image of x
19	πv	isomorphic coalition function
20	$X(N, v)$	set of preimputations
22	$v_{S,x}$	reduced coalition function
23	$\Gamma_{\mathcal{U}}, \Gamma_{\mathcal{U}}^{\mathcal{C}}$	set of all games, with nonempty cores

24	$\mathcal{P}(N)$	set of all pairs of players
27	2^N	set of all subsets
28	χ_S	characteristic vector
33	$x \cdot y$	scalar product
38	a_+	positive part of a
42	$\Gamma_{\mathcal{U}}^{tb}$	set of totally balanced games
45	(N, v, \mathcal{R})	TU game with coalition structure
46	Δ	set of TU games with coalition structures
47	$\Delta_{\mathcal{U}}, \Delta_{\mathcal{U}}^c$	set of all games, with nonempty cores
47	$\mathcal{P}(\mathcal{R})$	set of partners in \mathcal{R}
52	$\mathcal{T}_{k\ell}(N)$	coalitions containing k and not ℓ
53	$\mathcal{PM}(N, v, \mathcal{R})$	unconstrained bargaining set
55	$\mathcal{M}(N, v, \mathcal{R})$	bargaining set
57	$\ \cdot\ $	Euclidean norm
58	$e(S, x, v)$	excess of S at x
58	$s_{k\ell}(x, v)$	maximum surplus
65	$\mathcal{M}_r, \mathcal{PM}_r$	reactive (pre-)bargaining set
66	$\mathcal{M}_{sr}, \mathcal{PM}_{sr}$	semi-reactive (pre-)bargaining set
67	$\geq, >, \gg$	weak and strict inequalities (between vectors)
69	$\mathcal{MB}, (\mathcal{PMB})$	Mas-Colell (pre-)bargaining set
82	$\mathcal{PK}(N, v, \mathcal{R})$	prekernel
84	$\mathcal{N}(N, v, \mathcal{R})$	nucleolus of a game with coalition structure
84	$\mathcal{PN}(\dots), \nu(\dots)$	prenucleolus, point
87	$\mathcal{D}(\alpha, x, v)$	coalitions whose excess is at least α
89	$k \succ_v \ell$	desirability relation
95	$\mathcal{K}(N, v, \mathcal{R})$	kernel
110	$\binom{t}{k}$	binomial coefficient “ t choose k ”
113	\prod	(Cartesian) product
124	$\Psi(N, v, \mathcal{R})$	modiclus
133	$\mathcal{C}_{\varepsilon}(G)$	ε -core
134	$\mathcal{LC}(G)$	least-core
143	$[a, b]$	line segment between a and b
153	$\phi(v)$	Shapley value
159	$v_{S, \sigma}$	σ -reduced coalition function
171	$\phi_*(N, v, \mathcal{R})$	Aumann-Drèze value
173	$\bar{\phi}(N, v, \mathcal{R})$	Owen value
177	$\frac{\partial \bar{v}(\dots)}{\partial x^j}$	partial derivative
181	$\varphi : X \rightrightarrows Y$	set-valued function
183	\forall	universal quantification, “for all”
206	$V_{\alpha}(\cdot, \cdot)$	NTU coalition function of α -effectiveness
207	$V_{\beta}(\cdot, \cdot)$	NTU coalition function of β -effectiveness
210	(N, V)	NTU coalitional game
210	(N, V_v)	NTU game corresponding to TU game
217	∂Z	boundary of Z

224	$(S, V_{S,x})$	reduced NTU game
226	$\widehat{\Gamma}$	set of (non-levelled) NTU games
233	\exists	existential quantification, “there exists”
235	$\Delta_{++}(N)$	the interior of the unit simplex
235	Δ_{++}^V	set of viable vectors
239	$\Phi(N, V)$	set of Shapley NTU values
244	$\mathbf{x} = (x_S)_{S \in 2^N \setminus \{\emptyset\}}$	payoff configuration
245	$\Delta_{++}^{V(N)}$	set of viable vectors for $V(N)$
244	(S, V^S)	NTU subgame
247	σ	payoff configuration solution
247	$\Phi^{\mathcal{H}}$	Harsanyi solution
254	$\phi(N, V)$	consistent Shapley value of a hyperplane game
258	$\Phi^{MO}(N, V)$	set of consistent Shapley solutions
282	$\mathcal{PMB}^*(N, V)$	extended Mas-Colell bargaining set
295	$\mathcal{PM}^o, \mathcal{M}^o$	ordinal (pre-)bargaining set
306	$\mathcal{BCPK}(N, V, \mathcal{R})$	bilateral consistent prekernel

Introduction

This chapter is divided into three sections. In the first section the different kinds of cooperative games are discussed. A verbal description of the contents of this book is given in the second section and, finally, Section 1.3 describes one of the main goals of this book and comments on some related aspects.

1.1 Cooperative Games

This book is devoted to a study of the basic properties of solutions of cooperative games in coalitional form. Only Chapter 11 is an exception: In Sections 11.1 and 11.2 we study cooperative games in strategic form. The reason for this exception will be explained below. A coalitional or a strategic game is cooperative if the players can make binding agreements about the distribution of payoffs or the choice of strategies, even if these agreements are **not** specified or implied by the rules of the game (see Harsanyi and Selten (1988)). Binding agreements are prevalent in economics. Indeed, almost every one-stage seller-buyer transaction is binding. Moreover, most multi-stage seller-buyer transactions are supported by binding contracts. Usually, an agreement or a contract is binding if its violation entails high monetary penalties which deter the players from breaking it. However, agreements enforceable by a court may be more versatile.

Cooperative coalitional games are divided into two categories: games with transferable utilities and games with nontransferable utilities. We shall now consider these two classes of coalitional games in turn.

Let N be a set of players. A coalitional game with transferable utilities (a *TU game*) on N is a function that associates with each subset S of N (a *coalition*, if nonempty), a real number $v(S)$, the worth of S . Additionally, it is required that v assign zero to the empty set. If a coalition S forms, then it can divide its

worth, $v(S)$, in any possible way among its members. That is, S can achieve every payoff vector $x \in \mathbb{R}^S$ which is feasible, that is, which satisfies

$$\sum_{i \in S} x^i \leq v(S).$$

This is possible if money is available and desirable as a medium of exchange, and if the utilities of the players are linear in money (see Aumann (1960)).

Von Neumann and Morgenstern (1953) derive the TU coalition function from the strategic form of games with transferable utilities (i.e., utilities which are linear in money). The worth of a coalition S in a TU strategic game is its maximin value in the two-person zero-sum game, where S is opposed by its complement, $N \setminus S$, and correlated strategies of both S and $N \setminus S$ are used.

We consider the TU coalition function as a primitive concept, because in many applications of TU games coalition functions appear without any reference to a (TU) strategic game. This is, indeed, the case for many cost allocation problems. Furthermore, in a cooperative strategic game, any combination of strategies can be supported by a binding agreement. Hence the players focus on the choice of “stable” payoff vectors and not on the choice of a “stable” profile of strategies as in a noncooperative game. Clearly, the coalitional form is the suitable form for the analysis of the choice of a stable payoff distribution among the set of all feasible payoff distributions.

Coalitional games with nontransferable utilities (*NTU games*) were introduced in Aumann and Peleg (1960). They are suitable for the analysis of many cooperative and competitive phenomena in economics (see, e.g., Scarf (1967) and Debreu and Scarf (1963)). The axiomatic approach to NTU coalition functions, due to Aumann and Peleg (1960), has been motivated by a direct derivation of the NTU coalition function from the strategic form of the game. This approach is presented in Section 11.2.

1.2 Outline of the Book

We shall review the two parts consecutively.

1.2.1 TU Games

In Chapter 2 we first define coalitional TU games and some of their basic properties. Then we discuss market games, cost allocation games, and simple games. Games in the foregoing families frequently occur in applications. Finally, we systematically list the properties of the core. These properties,

suitably modified, serve later, in different combinations, as axioms for the core itself, the prekernel, the prenucleolus, and the Shapley value.

Chapter 3 is devoted to the core. The main results are:

- (1) A characterization of the set of all games with a nonempty core (the balanced games);
- (2) a characterization of market games as totally balanced games; and
- (3) an axiomatization of the core on the class of balanced games.

Various bargaining sets are studied in Chapter 4. We provide an existence theorem for bargaining sets which can be generalized to NTU games. Furthermore, it is proved that the Aumann-Davis-Maschler bargaining set of any convex game and of any assignment game coincides with its core.

Chapter 5 introduces the prekernel and the prenucleolus. We prove existence and uniqueness for the prenucleolus and, thereby, prove nonemptiness of the prekernel and reconfirm the nonemptiness of the aforementioned bargaining sets. The prekernel is axiomatized in Section 5.4. Moreover, we investigate individual rationality for the prekernel and, in addition, prove that it is reasonable. Finally, we prove that the kernel of a convex game coincides with its nucleolus.

Chapter 6 mainly focuses on:

- (1) Sobolev's axiomatization of the prenucleolus;
- (2) an investigation of the nucleolus of strong weighted majority games which shows, in particular, that the nucleolus of a strong weighted majority game is a representation of the game; and
- (3) definition and verification of the basic properties of the modiclus; in particular, we show that the modiclus of any weighted majority game is a representation of the game.

In Chapter 7, ε -cores and the least-core are introduced, and their intuitive properties are studied. The main results are:

- (1) A geometric characterization of the intersection of the prekernel of a game with an ε -core; and
- (2) an algorithm for computing the prenucleolus.

Chapter 8 is entirely devoted to the Shapley value. Four axiomatizations of the Shapley value are presented:

- (1) Shapley's axiomatization using additivity;

- (2) Young's axiomatization using strong monotonicity;
- (3) an axiomatization based on consistency by Hart and Mas-Colell; and
- (4) Sobolev's axiomatization based on a special reduced game.

Moreover, Dubey's axiomatization of the Shapley value on the set of monotonic simple games is presented. We conclude with Owen's value of games with a priori unions and his formula relating the Shapley value of a game to the multilinear extension of the game.

Chapter 9 is devoted to continuity properties of solutions. All our solutions are upper hemicontinuous and closed-valued. The core and the nucleolus are actually continuous. The continuity of the Shapley value is obvious.

In Chapter 10 dynamic systems for the prekernel and various bargaining sets are introduced. Some results on stability and local asymptotic stability are obtained.

1.2.2 NTU Games

In Chapter 11 we define cooperative games in strategic form and derive their coalitional games. This serves as a basis for the axiomatic definition of coalitional NTU games.

Chapter 12 is entirely devoted to the core of NTU games. First we prove that suitably balanced NTU games have a nonempty core. Then we show that convex NTU games have a nonempty core. We conclude with various axiomatizations of the core.

In Chapter 13 we provide existence proofs and characterizations for the Shapley NTU value and the Harsanyi solution. We also give an axiomatic characterization of each solution.

Chapter 14 is devoted to the consistent Shapley value. First we investigate hyperplane games following Maschler and Owen (1989). Then we prove existence of the consistent value for p -smooth games. We conclude with an axiomatic analysis of the consistent value.

Chapter 15 investigates the classical and Mas-Colell bargaining sets for NTU games. We deal mainly with (NTU) majority voting games. We show that if there are at most five alternatives, then the Mas-Colell bargaining is nonempty. For majority games with six or more alternatives the Mas-Colell set may be empty. Using more elaborated examples we show that the Mas-Colell bargaining set of a non-levelled superadditive game may be empty. We conclude with some limit theorems for bargaining sets of majority games.

In Chapter 16 we conclude with an existence proof for the ordinal bargaining set of NTU games and with a discussion of related solutions.

1.2.3 A Guide for the Reader

We should like to make the following remarks.

Remark 1.2.1. The investigations of the various solutions are almost independent of each other. For example, you may study the core by reading Chapters 3 and 12 and browsing Sections 2.3 and 11.3. If you are interested only in the Shapley value, you should read Chapter 8 and Sections 13.1 and 13.2. Similar possibilities exist for the bargaining set, kernel, and nucleolus (see the Table of Contents).

Remark 1.2.2. If you plan an introductory course on game theory, then you may use Chapters 2, 3, and 8 for introducing cooperative games at the end of your course.

Remark 1.2.3. Chapters 2 - 12 may be used for a one-semester course on cooperative games. Part II may be used in a graduate course on cooperative games without side-payments.

Remark 1.2.4. Each section concludes with some exercises. The reader is advised to solve at least those exercises that are used in the text to complete the proofs of various results.

1.3 Special Remarks

The analysis of solutions of cooperative games emphasizes the axiomatic approaches which do not rely on interpersonal comparisons of utility. Moreover, we comment on the Nash program.

1.3.1 Axiomatizations

One of our main goals is to supply uniform and coherent axiomatizations for the main solutions of cooperative games. Indeed, this book is the first to include axiomatizations of the core, the prekernel, and the prenucleolus. Every axiom which we use is satisfied, sometimes after a suitable modification, by the core of TU games; the only exception is consistency (in the sense of Hart and Mas-Colell), which is satisfied only by the Shapley value. Table 8.11.1 shows our success for TU games.

1.3.2 Interpersonal Comparisons of Utility

For a definition of interpersonal comparisons of utility the reader is referred to Harsanyi (1992). In our view a solution is free of interpersonal comparisons

of utility, if it has an axiomatization which does not use interpersonal comparisons of utility. As none of our axioms implies interpersonal comparisons of utility, all the solutions which we discuss do not rely on interpersonal comparisons of utility. (Covariance for TU games implies cardinal unit comparability. However, it is **not** used for actual comparisons of utilities (see Luce and Raiffa (1957), pp. 168 - 169).) The bargaining set, which is left unaxiomatized, does not involve interpersonal comparisons of utility by its definition.

1.3.3 Nash's Program

According to Harsanyi and Selten (1988), Section 1.11, "... analysis of any cooperative game G should be based on a formal bargaining model $B(G)$, involving bargaining moves and countermoves by the various players and resulting in an agreement about the outcome of the game. Formally, this bargaining model $B(G)$ would always be a noncooperative game in extensive form (or possibly in normal form), and the solution of the cooperative game G would be defined in terms of the equilibrium points of this noncooperative game $B(G)$." This claim is known as Nash's program. Peleg (1996) and (1997) shows that Nash's program cannot be implemented. Hence, we shall not further discuss it.

Part I

TU Games

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