THEORY AND DECISION LIBRARY

SERIES C: GAME THEORY, MATHEMATICAL PROGRAMMING AND OPERATIONS RESEARCH

Bezalel Peleg
Peter Sudhölter

INTRODUCTION TO THE THEORY OF COOPERATIVE GAMES

SECOND EDITION

THEORY AND DECISION LIBRARY

General Editors: W. Leinfellner (*Vienna*) and G. Eberlein (*Munich*)

Series A: Philosophy and Methodology of the Social Sciences

Series B: Mathematical and Statistical Methods

Series C: Game Theory, Mathematical Programming and Operations Research

Series D: System Theory, Knowledge Engineering and Problem Solving

SERIES C: GAME THEORY, MATHEMATICAL PROGRAMMING AND OPERATIONS RESEARCH

VOLUME 34

Scope: Particular attention is paid in this series to game theory and operations research, their formal aspects and their applications to economic, political and social sciences as well as to sociobiology. It will encourage high standards in the application of game-theoretical methods to individual and social decision making.

The titles published in this series are listed at the end of this volume.

Bezalel Peleg · Peter Sudhölter

Introduction to the Theory of Cooperative Games

Second Edition

Springer

Professor Bezalel Peleg
The Hebrew University of Jerusalem
Institute of Mathematics and
Center for the Study of Rationality
Givat-Ram, Feldman Building
91904 Jerusalem
Israel
pelegba@math.huji.ac.il

Professor Peter Sudhölter
University of Southern Denmark
Department of Business and Economics
Campusvej 55
5230 Odense M
Denmark
psu@sam.sdu.dk

Library of Congress Control Number: 2007931451

ISSN 0924-6126
ISBN 978-3-540-72944-0 Springer Berlin Heidelberg New York
ISBN 978-1-4020-7410-3 1st Edition Springer Berlin Heidelberg New York

Springer is a part of Springer Science+Business Media

springer.com

Production: LE-TEX Jelonek, Schmidt & Vöckler GbR, Leipzig, Germany
Cover design: WMX Design GmbH, Heidelberg

Spin 12073665 Printed on acid-free paper 43/3180/YL - 5 4 3 2 1 0

Preface to the Second Edition

The main purpose of the second edition is to enhance and expand the treatment of games with nontransferable utility. The main changes are:

(1) Chapter 13 is devoted entirely to the Shapley value and the Harsanyi solution. Section 13.4 is new and contains an axiomatization of the Harsanyi solution.

(2) Chapter 14 deals exclusively with the consistent Shapley value. Sections 14.2 and 14.3 are new and present an existence proof for the consistent value and an axiomatization of the consistent value respectively. Section 14.1, which was part of the old Chapter 13, deals with the consistent value of polyhedral games.

(3) Chapter 15 is almost entirely new. It is mainly devoted to an investigation of the Mas-Colell bargaining set of majority voting games. The existence of the Mas-Colell set is investigated and various limit theorems are proved for majority voting games. As a corollary of our results we show the existence of a four-person super-additive and non-levelled (NTU) game whose Mas-Colell bargaining set is empty.

(4) The treatment of the ordinal bargaining set was moved to the final chapter 16.

We also have used this opportunity to remove typos and inaccuracies from Chapters 2 – 12 which otherwise remained intact.

We are indebted to all our readers who pointed out some typo. In particular we thank Michael Maschler for his comments and Martina Bihn who personally supported this edition.

June 2007 Bezalel Peleg and Peter Sudhölter

Preface to the First Edition

In this book we study systematically the main solutions of cooperative games: the core, bargaining set, kernel, nucleolus, and the Shapley value of TU games, and the core, the Shapley value, and the ordinal bargaining set of NTU games. To each solution we devote a separate chapter wherein we study its properties in full detail. Moreover, important variants are defined or even intensively analyzed. We also investigate in separate chapters continuity, dynamics, and geometric properties of solutions of TU games. Our study culminates in uniform and coherent axiomatizations of all the foregoing solutions (excluding the bargaining set).

It is our pleasure to acknowledge the help of the following persons and institutions. We express our gratitude to Michael Maschler for his detailed comments on an early version, due to the first author, of Chapters 2 – 8. We thank Michael Borns for the linguistic edition of the manuscript of this book. We are indebted to Claus-Jochen Haake, Sven Klauke, and Christian Weiß for reading large parts of the manuscript and suggesting many improvements. Peter Sudhölter is grateful to the Center for Rationality and Interactive Decision Theory of the Hebrew University of Jerusalem and to the Edmund Landau Center for Research in Mathematical Analysis and Related Areas, the Institute of Mathematics of the Hebrew University of Jerusalem, for their hospitality during the academic year 2000-01 and during the summer of 2002. These institutions made the typing of the manuscript possible. He is also grateful to the Institute of Mathematical Economics, University of Bielefeld, for its support during several visits in the years 2001 and 2002.

December 2002 Bezalel Peleg and Peter Sudhölter

Contents

Preface to the Second Edition V

Preface to the First Edition VI

List of Figures XIII

List of Tables XV

Notation and Symbols XVII

1 Introduction 1
- 1.1 Cooperative Games 1
- 1.2 Outline of the Book 2
  - 1.2.1 TU Games 2
  - 1.2.2 NTU Games 4
  - 1.2.3 A Guide for the Reader 5
- 1.3 Special Remarks 5
  - 1.3.1 Axiomatizations 5
  - 1.3.2 Interpersonal Comparisons of Utility 5
  - 1.3.3 Nash's Program 6

Part I TU Games

2 Coalitional TU Games and Solutions 9
- 2.1 Coalitional Games 9

2.2 Some Families of Games ... 13
2.2.1 Market Games ... 13
2.2.2 Cost Allocation Games ... 14
2.2.3 Simple Games ... 16
2.3 Properties of Solutions ... 19
2.4 Notes and Comments ... 26

3 The Core ... 27
3.1 The Bondareva-Shapley Theorem ... 27
3.2 An Application to Market Games ... 32
3.3 Totally Balanced Games ... 34
3.4 Some Families of Totally Balanced Games ... 35
3.4.1 Minimum Cost Spanning Tree Games ... 35
3.4.2 Permutation Games ... 36
3.5 A Characterization of Convex Games ... 39
3.6 An Axiomatization of the Core ... 40
3.7 An Axiomatization of the Core on Market Games ... 42
3.8 The Core for Games with Various Coalition Structures ... 44
3.9 Notes and Comments ... 48

4 Bargaining Sets ... 51
4.1 The Bargaining Set $\mathcal{M}$ ... 52
4.2 Existence of the Bargaining Set ... 57
4.3 Balanced Superadditive Games and the Bargaining Set ... 62
4.4 Further Bargaining Sets ... 64
4.4.1 The Reactive and the Semi-reactive Bargaining Set ... 65
4.4.2 The Mas-Colell Bargaining Set ... 69
4.5 Non-monotonicity of Bargaining Sets ... 72
4.6 The Bargaining Set and Syndication: An Example ... 76
4.7 Notes and Comments ... 79

5 The Prekernel, Kernel, and Nucleolus 81
5.1 The Nucleolus and the Prenucleolus 82
5.2 The Reduced Game Property 86
5.3 Desirability, Equal Treatment, and the Prekernel 89
5.4 An Axiomatization of the Prekernel 91
5.5 Individual Rationality and the Kernel 94
5.6 Reasonableness of the Prekernel and the Kernel 98
5.7 The Prekernel of a Convex Game 100
5.8 The Prekernel and Syndication 103
5.9 Notes and Comments 105

6 The Prenucleolus 107
6.1 A Combinatorial Characterization of the Prenucleolus 108
6.2 Preliminary Results 109
6.3 An Axiomatization of the Prenucleolus 112
6.3.1 An Axiomatization of the Nucleolus 115
6.3.2 The Positive Core 117
6.4 The Prenucleolus of Games with Coalition Structures 119
6.5 The Nucleolus of Strong Weighted Majority Games 120
6.6 The Modiclus 124
6.6.1 Constant-Sum Games 129
6.6.2 Convex Games 130
6.6.3 Weighted Majority Games 131
6.7 Notes and Comments 132

7 Geometric Properties of the ε-Core, Kernel, and Prekernel 133
7.1 Geometric Properties of the ε-Core 133
7.2 Some Properties of the Least-Core 136
7.3 The Reasonable Set 138
7.4 Geometric Characterizations of the Prekernel and Kernel 142
7.5 A Method for Computing the Prenucleolus 146
7.6 Notes and Comments 149

8 The Shapley Value 151
8.1 Existence and Uniqueness of the Value 152
8.2 Monotonicity Properties of Solutions and the Value 156
8.3 Consistency 159
8.4 The Potential of the Shapley Value 161
8.5 A Reduced Game for the Shapley Value 163
8.6 The Shapley Value for Simple Games 168
8.7 Games with Coalition Structures 170
8.8 Games with A Priori Unions 172
8.9 Multilinear Extensions of Games 175
8.10 Notes and Comments 178
8.11 A Summary of Some Properties of the Main Solutions 179

9 Continuity Properties of Solutions 181
9.1 Upper Hemicontinuity of Solutions 181
9.2 Lower Hemicontinuity of Solutions 184
9.3 Continuity of the Prenucleolus 187
9.4 Notes and Comments 188

10 Dynamic Bargaining Procedures for the Kernel and the Bargaining Set 189
10.1 Dynamic Systems for the Kernel and the Bargaining Set 190
10.2 Stable Sets of the Kernel and the Bargaining Set 195
10.3 Asymptotic Stability of the Nucleolus 198
10.4 Notes and Comments 199

Part II NTU Games

11 Cooperative Games in Strategic and Coalitional Form 203
11.1 Cooperative Games in Strategic Form 203
11.2 α- and β-Effectiveness 205
11.3 Coalitional Games with Nontransferable Utility 209

11.4 Cooperative Games with Side Payments but Without Transferable Utility 210

11.5 Notes and Comments 212

12 The Core of NTU Games 213

12.1 Individual Rationality, Pareto Optimality, and the Core 214

12.2 Balanced NTU Games 215

12.3 Ordinal and Cardinal Convex Games 220

12.3.1 Ordinal Convex Games 220

12.3.2 Cardinal Convex Games 222

12.4 An Axiomatization of the Core 224

12.4.1 Reduced Games of NTU Games 224

12.4.2 Axioms for the Core 226

12.4.3 Proof of Theorem 12.4.8 227

12.5 Additional Properties and Characterizations 230

12.6 Notes and Comments 233

13 The Shapley NTU Value and the Harsanyi Solution 235

13.1 The Shapley Value of NTU Games 235

13.2 A Characterization of the Shapley NTU Value 239

13.3 The Harsanyi Solution 243

13.4 A Characterization of the Harsanyi Solution 247

13.5 Notes and Comments 251

14 The Consistent Shapley Value 253

14.1 For Hyperplane Games 253

14.2 For p-Smooth Games 257

14.3 Axiomatizations 261

14.3.1 The Role of IIA 264

14.3.2 Logical Independence 265

14.4 Notes and Comments 267

15 On the Classical Bargaining Set and the Mas-Colell Bargaining Set for NTU Games 269

15.1 Preliminaries 270

15.1.1 The Bargaining Set $\mathcal{M}$ 270

15.1.2 The Mas-Colell Bargaining Set $\mathcal{MB}$ and Majority Voting Games 272

15.1.3 The 3×3 Voting Paradox 274

15.2 Voting Games with an Empty Mas-Colell Bargaining Set 278

15.3 Non-levelled NTU Games with an Empty Mas-Colell Prebargaining Set 282

15.3.1 The Example 283

15.3.2 Non-levelled Games 286

15.4 Existence Results for Many Voters 289

15.5 Notes and Comments 292

16 Variants of the Davis-Maschler Bargaining Set for NTU Games 295

16.1 The Ordinal Bargaining Set $\mathcal{M}^o$ 295

16.2 A Proof of Billera's Theorem 299

16.3 Solutions Related to $\mathcal{M}^o$ 302

16.3.1 The Ordinal Reactive and the Ordinal Semi-Reactive Bargaining Sets 302

16.3.2 Solutions Related to the Prekernel 303

16.4 Notes and Comments 308

References 311

Author Index 321

Subject Index 323

List of Figures

Fig. 2.2.1 Connection Cost 15

Fig. 4.4.1 The Projective Seven-Person Game 66

Fig. 13.1.1 The Shapley Value 237

List of Tables

Table 8.11.1 Solutions and Properties 179

Table 15.1.1 Preference Profile of a 4-Person Voting Problem 274

Table 15.1.2 Preference Profile of the 3×3 Voting Paradox 275

Table 15.1.3 Preference Profile of a 4-Alternative Voting Problem . . . 276

Table 15.2.4 Preference Profile leading to an empty $\mathcal{PMB}$ 278

Table 15.3.5 Preference Profile on 10 Alternatives 283

Table 15.3.6 Domination Relation 283

Table 15.3.7 Constructions of Strong Objections 284

Notation and Symbols

We shall now list some of our notation.

The field of real numbers is denoted by $\mathbb{R}$ and $\mathbb{R}_+$ is the set of nonnegative reals. For a finite set S, the Euclidean vector space of real functions with the domain S is denoted by $\mathbb{R}^S$. An element x of $\mathbb{R}^S$ is represented by the vector $(x^i)_{i \in S}$. Also, $\mathbb{R}^S_+ = \{x \in \mathbb{R}^S \mid x^i \geq 0 \text{ for all } i \in S\}$ and $\mathbb{R}^S_{++} = \{x \in \mathbb{R}^S \mid x^i > 0 \text{ for all } i \in S\}$. If $x, y \in \mathbb{R}^N$, $S, T \subseteq N$, and $S \cap T = \emptyset$, then $x^S = (x^i)_{i \in S}$ and $z = (x^S, y^T) \in \mathbb{R}^{S \cup T}$ is given by $z^i = x^i$ for all $i \in S$ and $z^j = y^j$ for all $j \in T$.

The symbols in the following list are ordered according to the page numbers, the numbers in the first column, of their definitions or first occurrences.

2	$X \setminus Y$	set difference ($\{x \in X \mid x \notin Y\}$)
9	$\mathcal{U}$	the universe of players
9	(N, v)	coalitional TU game
10	$\Rightarrow$	implies, implication
11	$x(S)$	aggregate amount of S
11	$\alpha v + \beta$	strategically equivalent coalition function to v
12	$\mathcal{SYM}$	group of symmetries
12	$\|A\|$	cardinality of A
19	$X^*(N, v)$	feasible payoff vectors
19	$\mathit{\Gamma}$	set of TU games
19	σ	solution
19	$\mathcal{C}(N, v)$	the core
19	$\pi(x)$	image of x
19	πv	isomorphic coalition function
20	$X(N, v)$	set of preimputations
22	$v_{S,x}$	reduced coalition function
23	$\mathit{\Gamma}_{\mathcal{U}}, \mathit{\Gamma}^{\mathcal{C}}_{\mathcal{U}}$	set of all games, with nonempty cores

24	$\mathcal{P}(N)$	set of all pairs of players
27	2^N	set of all subsets
28	χ_S	characteristic vector
33	$x \cdot y$	scalar product
38	a_+	positive part of a
42	$\Gamma_{\mathcal{U}}^{tb}$	set of totally balanced games
45	$(N, v, \mathcal{R})$	TU game with coalition structure
46	Δ	set of TU games with coalition structures
47	$\Delta_{\mathcal{U}}, \Delta_{\mathcal{U}}^{\mathcal{C}}$	set of all games, with nonempty cores
47	$\mathcal{P}(\mathcal{R})$	set of partners in $\mathcal{R}$
52	$\mathcal{T}_{k\ell}(N)$	coalitions containing k and not ℓ
53	$\mathcal{PM}(N, v, \mathcal{R})$	unconstrained bargaining set
55	$\mathcal{M}(N, v, \mathcal{R})$	bargaining set
57	$\Vert \cdot \Vert$	Euclidean norm
58	$e(S, x, v)$	excess of S at x
58	$s_{k\ell}(x, v)$	maximum surplus
65	$\mathcal{M}_r, \mathcal{PM}_r$	reactive (pre-)bargaining set
66	$\mathcal{M}_{sr}, \mathcal{PM}_{sr}$	semi-reactive (pre-)bargaining set
67	$\geq, >, \gg$	weak and strict inequalities (between vectors)
69	$\mathcal{MB}, (\mathcal{PMB})$	Mas-Colell (pre-)bargaining set
82	$\mathcal{PK}(N, v, \mathcal{R})$	prekernel
84	$\mathcal{N}(N, v, \mathcal{R})$	nucleolus of a game with coalition structure
84	$\mathcal{PN}(\cdots), \nu(\cdots)$	prenucleolus, point
87	$\mathcal{D}(\alpha, x, v)$	coalitions whose excess is at least α
89	$k \succsim_v \ell$	desirability relation
95	$\mathcal{K}(N, v, \mathcal{R})$	kernel
110	$\binom{t}{k}$	binomial coefficient "t choose k"
113	$\prod$	(Cartesian) product
124	$\Psi(N, v, \mathcal{R})$	modiclus
133	$\mathcal{C}_\varepsilon(G)$	ε-core
134	$\mathcal{LC}(G)$	least-core
143	$[a, b]$	line segment between a and b
153	$\phi(v)$	Shapley value
159	$v_{S,\sigma}$	σ-reduced coalition function
171	$\phi_*(N, v, \mathcal{R})$	Aumann-Drèze value
173	$\overline{\phi}(N, v, \mathcal{R})$	Owen value
177	$\frac{\partial \overline{v}(\cdots)}{\partial x^j}$	partial derivative
181	$\varphi : X \rightrightarrows Y$	set-valued function
183	$\forall$	universal quantification, "for all"
206	$V_\alpha(\cdot, \cdot)$	NTU coalition function of α-effectiveness
207	$V_\beta(\cdot, \cdot)$	NTU coalition function of β-effectiveness
210	(N, V)	NTU coalitional game
210	(N, V_v)	NTU game corresponding to TU game
217	∂Z	boundary of Z

224	$(S, V_{S,x})$	reduced NTU game
226	$\widehat{\Gamma}$	set of (non-levelled) NTU games
233	$\exists$	existential quantification, "there exists"
235	$\Delta_{++}(N)$	the interior of the unit simplex
235	Δ_{++}^{V}	set of viable vectors
239	$\Phi(N, V)$	set of Shapley NTU values
244	$\boldsymbol{x} = (x_S)_{S \in 2^N \setminus \{\emptyset\}}$	payoff configuration
245	$\Delta_{++}^{V(N)}$	set of viable vectors for $V(N)$
244	(S, V^S)	NTU subgame
247	$\boldsymbol{\sigma}$	payoff configuration solution
247	$\boldsymbol{\Phi}^{\mathcal{H}}$	Harsanyi solution
254	$\phi(N, V)$	consistent Shapley value of a hyperplane game
258	$\boldsymbol{\Phi}^{\mathcal{MO}}(N, V)$	set of consistent Shapley solutions
282	$\mathcal{P}MB^{*}(N, V)$	extended Mas-Colell bargaining set
295	$\mathcal{PM}^{o}, \mathcal{M}^{o}$	ordinal (pre-)bargaining set
306	$\mathcal{BCPK}(N, V, \mathcal{R})$	bilateral consistent prekernel

1

Introduction

This chapter is divided into three sections. In the first section the different kinds of cooperative games are discussed. A verbal description of the contents of this book is given in the second section and, finally, Section 1.3 describes one of the main goals of this book and comments on some related aspects.

1.1 Cooperative Games

This book is devoted to a study of the basic properties of solutions of cooperative games in coalitional form. Only Chapter 11 is an exception: In Sections 11.1 and 11.2 we study cooperative games in strategic form. The reason for this exception will be explained below. A coalitional or a strategic game is cooperative if the players can make binding agreements about the distribution of payoffs or the choice of strategies, even if these agreements are **not** specified or implied by the rules of the game (see Harsanyi and Selten (1988)). Binding agreements are prevalent in economics. Indeed, almost every one-stage seller-buyer transaction is binding. Moreover, most multi-stage seller-buyer transactions are supported by binding contracts. Usually, an agreement or a contract is binding if its violation entails high monetary penalties which deter the players from breaking it. However, agreements enforceable by a court may be more versatile.

Cooperative coalitional games are divided into two categories: games with transferable utilities and games with nontransferable utilities. We shall now consider these two classes of coalitional games in turn.

Let N be a set of players. A coalitional game with transferable utilities (a *TU game*) on N is a function that associates with each subset S of N (a *coalition*, if nonempty), a real number $v(S)$, the worth of S. Additionally, it is required that v assign zero to the empty set. If a coalition S forms, then it can divide its worth, $v(S)$, in any possible way among its members. That is, S can achieve every payoff vector $x \in \mathbb{R}^S$ which is feasible, that is, which satisfies

$$\sum_{i \in S} x^i \leq v(S).$$

This is possible if money is available and desirable as a medium of exchange, and if the utilities of the players are linear in money (see Aumann (1960)).

Von Neumann and Morgenstern (1953) derive the TU coalition function from the strategic form of games with transferable utilities (i.e., utilities which are linear in money). The worth of a coalition S in a TU strategic game is its maximin value in the two-person zero-sum game, where S is opposed by its complement, $N \setminus S$, and correlated strategies of both S and $N \setminus S$ are used.

We consider the TU coalition function as a primitive concept, because in many applications of TU games coalition functions appear without any reference to a (TU) strategic game. This is, indeed, the case for many cost allocation problems. Furthermore, in a cooperative strategic game, any combination of strategies can be supported by a binding agreement. Hence the players focus on the choice of "stable" payoff vectors and not on the choice of a "stable" profile of strategies as in a noncooperative game. Clearly, the coalitional form is the suitable form for the analysis of the choice of a stable payoff distribution among the set of all feasible payoff distributions.

Coalitional games with nontransferable utilities (*NTU games*) were introduced in Aumann and Peleg (1960). They are suitable for the analysis of many cooperative and competitive phenomena in economics (see, e.g., Scarf (1967) and Debreu and Scarf (1963)). The axiomatic approach to NTU coalition functions, due to Aumann and Peleg (1960), has been motivated by a direct derivation of the NTU coalition function from the strategic form of the game. This approach is presented in Section 11.2.

1.2 Outline of the Book

We shall review the two parts consecutively.

1.2.1 TU Games

In Chapter 2 we first define coalitional TU games and some of their basic properties. Then we discuss market games, cost allocation games, and simple games. Games in the foregoing families frequently occur in applications. Finally, we systematically list the properties of the core. These properties, suitably modified, serve later, in different combinations, as axioms for the core itself, the prekernel, the prenucleolus, and the Shapley value.

Chapter 3 is devoted to the core. The main results are:

(1) A characterization of the set of all games with a nonempty core (the balanced games);

(2) a characterization of market games as totally balanced games; and

(3) an axiomatization of the core on the class of balanced games.

Various bargaining sets are studied in Chapter 4. We provide an existence theorem for bargaining sets which can be generalized to NTU games. Furthermore, it is proved that the Aumann-Davis-Maschler bargaining set of any convex game and of any assignment game coincides with its core.

Chapter 5 introduces the prekernel and the prenucleolus. We prove existence and uniqueness for the prenucleolus and, thereby, prove nonemptiness of the prekernel and reconfirm the nonemptiness of the aforementioned bargaining sets. The prekernel is axiomatized in Section 5.4. Moreover, we investigate individual rationality for the prekernel and, in addition, prove that it is reasonable. Finally, we prove that the kernel of a convex game coincides with its nucleolus.

Chapter 6 mainly focuses on:

(1) Sobolev's axiomatization of the prenucleolus;

(2) an investigation of the nucleolus of strong weighted majority games which shows, in particular, that the nucleolus of a strong weighted majority game is a representation of the game; and

(3) definition and verification of the basic properties of the modiclus; in particular, we show that the modiclus of any weighted majority game is a representation of the game.

In Chapter 7, ε-cores and the least-core are introduced, and their intuitive properties are studied. The main results are:

(1) A geometric characterization of the intersection of the prekernel of a game with an ε-core; and

(2) an algorithm for computing the prenucleolus.

Chapter 8 is entirely devoted to the Shapley value. Four axiomatizations of the Shapley value are presented:

(1) Shapley's axiomatization using additivity;

(2) Young's axiomatization using strong monotonicity;

(3) an axiomatization based on consistency by Hart and Mas-Colell; and

(4) Sobolev's axiomatization based on a special reduced game.

Moreover, Dubey's axiomatization of the Shapley value on the set of monotonic simple games is presented. We conclude with Owen's value of games with a priori unions and his formula relating the Shapley value of a game to the multilinear extension of the game.

Chapter 9 is devoted to continuity properties of solutions. All our solutions are upper hemicontinuous and closed-valued. The core and the nucleolus are actually continuous. The continuity of the Shapley value is obvious.

In Chapter 10 dynamic systems for the prekernel and various bargaining sets are introduced. Some results on stability and local asymptotic stability are obtained.

1.2.2 NTU Games

In Chapter 11 we define cooperative games in strategic form and derive their coalitional games. This serves as a basis for the axiomatic definition of coalitional NTU games.

Chapter 12 is entirely devoted to the core of NTU games. First we prove that suitably balanced NTU games have a nonempty core. Then we show that convex NTU games have a nonempty core. We conclude with various axiomatizations of the core.

In Chapter 13 we provide existence proofs and characterizations for the Shapley NTU value and the Harsanyi solution. We also give an axiomatic characterization of each solution.

Chapter 14 is devoted to the consistent Shapley value. First we investigate hyperplane games following Maschler and Owen (1989). Then we prove existence of the consistent value for p-smooth games. We conclude with an axiomatic analysis of the consistent value.

Chapter 15 investigates the classical and Mas-Colell bargaining sets for NTU games. We deal mainly with (NTU) majority voting games. We show that if there are at most five alternatives, then the Mas-Colell bargaining is nonempty. For majority games with six or more alternatives the Mas-Colell set may be empty. Using more elaborated examples we show that the Mas-Colell bargaining set of a non-levelled superadditive game may be empty. We conclude with some limit theorems for bargaining sets of majority games.

In Chapter 16 we conclude with an existence proof for the ordinal bargaining set of NTU games and with a discussion of related solutions.

1.2.3 A Guide for the Reader

We should like to make the following remarks.

Remark 1.2.1. The investigations of the various solutions are almost independent of each other. For example, you may study the core by reading Chapters 3 and 12 and browsing Sections 2.3 and 11.3. If you are interested only in the Shapley value, you should read Chapter 8 and Sections 13.1 and 13.2. Similar possibilities exist for the bargaining set, kernel, and nucleolus (see the Table of Contents).

Remark 1.2.2. If you plan an introductory course on game theory, then you may use Chapters 2, 3, and 8 for introducing cooperative games at the end of your course.

Remark 1.2.3. Chapters 2 - 12 may be used for a one-semester course on cooperative games. Part II may be used in a graduate course on cooperative games without side-payments.

Remark 1.2.4. Each section concludes with some exercises. The reader is advised to solve at least those exercises that are used in the text to complete the proofs of various results.

1.3 Special Remarks

The analysis of solutions of cooperative games emphasizes the axiomatic approaches which do not rely on interpersonal comparisons of utility. Moreover, we comment on the Nash program.

1.3.1 Axiomatizations

One of our main goals is to supply uniform and coherent axiomatizations for the main solutions of cooperative games. Indeed, this book is the first to include axiomatizations of the core, the prekernel, and the prenucleolus. Every axiom which we use is satisfied, sometimes after a suitable modification, by the core of TU games; the only exception is consistency (in the sense of Hart and Mas-Colell), which is satisfied only by the Shapley value. Table 8.11.1 shows our success for TU games.

1.3.2 Interpersonal Comparisons of Utility

For a definition of interpersonal comparisons of utility the reader is referred to Harsanyi (1992). In our view a solution is free of interpersonal comparisons of utility, if it has an axiomatization which does not use interpersonal comparisons of utility. As none of our axioms implies interpersonal comparisons of utility, all the solutions which we discuss do not rely on interpersonal comparisons of utility. (Covariance for TU games implies cardinal unit comparability. However, it is **not** used for actual comparisons of utilities (see Luce and Raiffa (1957), pp. 168 - 169).) The bargaining set, which is left unaxiomatized, does not involve interpersonal comparisons of utility by its definition.

1.3.3 Nash's Program

According to Harsanyi and Selten (1988), Section 1.11, "… analysis of any cooperative game G should be based on a formal bargaining model $B(G)$, involving bargaining moves and countermoves by the various players and resulting in an agreement about the outcome of the game. Formally, this bargaining model $B(G)$ would always be a noncooperative game in extensive form (or possibly in normal form), and the solution of the cooperative game G would be defined in terms of the equilibrium points of this noncooperative game $B(G)$." This claim is known as Nash's program. Peleg (1996) and (1997) shows that Nash's program cannot be implemented. Hence, we shall not further discuss it.

Part I

TU Games