



fifth edition

# MATHEMATICS FOR ECONOMICS AND BUSINESS

IAN JACQUES

**FT** Prentice Hall  
FINANCIAL TIMES

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**To my mother, and in memory of my father**

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# Preface

This book is intended primarily for students on economics, business studies and management courses. It assumes very little prerequisite knowledge, so it can be read by students who have not undertaken a mathematics course for some time. The style is informal and the book contains a large number of worked examples. Students are encouraged to tackle problems for themselves as they read through each section. Detailed solutions are provided so that all answers can be checked. Consequently, it should be possible to work through this book on a self-study basis. The material is wide ranging, and varies from elementary topics such as percentages and linear equations, to more sophisticated topics such as constrained optimization of multivariate functions. The book should therefore be suitable for use on both low- and high-level quantitative methods courses. Examples and exercises are included which make use of the computer software packages Excel and Maple.

This book was first published in 1991. The prime motivation for writing it then was to try and produce a textbook that students could actually read and understand for themselves. This remains the guiding principle and the most significant change for this, the fifth edition, is in the design, rather than content. I was brought up with the fixed idea that mathematics textbooks were written in a small font with many equations crammed on to a page. However, I fully accept that these days books need to look attractive and be easy to negotiate. I hope that the new style will encourage more students to read it and will reduce the ‘fear factor’ of mathematics. In response to anonymous reviewers’ comments, I have included additional problems for several exercises together with two new appendices on implicit differentiation and Hessian matrices. Finally, I have also included the highlighted key terms at the end of each section and in a glossary at the end of the book.

The book now has an accompanying website that is intended to be rather more than just a gimmick. I hope that the commentary in the Instructor’s Manual will help tutors using the book for the first time. It also contains about a hundred new questions. Although a few of these problems are similar to those in the main book, the majority of questions are genuinely different. There are roughly two test exercises per chapter, which are graded to accommodate different levels of student abilities. These are provided on the website so that they can easily be cut, pasted and edited to suit. Fully worked solutions and marking schemes are included. Tutors can also control access. The website has a section containing multiple-choice tests. These can be given to students for further practice or used for assessment. The multiple choice questions can be marked online with the results automatically transferred to the tutor’s markbook if desired.

**Ian Jacques**





# Introduction

## Getting Started

### Notes for students: how to use this book

I am always amazed by the mix of students on first-year economics courses. Some have not acquired any mathematical knowledge beyond elementary algebra (and even that can be of a rather dubious nature), some have never studied economics before in their lives, while others have passed preliminary courses in both. Whatever category you are in, I hope that you will find this book of value. The chapters covering algebraic manipulation, simple calculus, finance and matrices should also benefit students on business studies and accountancy courses.

The first few chapters are aimed at complete beginners and students who have not taken mathematics courses for some time. I would like to think that these students once enjoyed mathematics and had every intention of continuing their studies in this area, but somehow never found the time to fit it into an already overcrowded academic timetable. However, I suspect that the reality is rather different. Possibly they hated the subject, could not understand it and dropped it at the earliest opportunity. If you find yourself in this position, you are probably horrified to discover that you must embark on a quantitative methods course with an examination looming on the horizon. However, there is no need to worry. My experience is that every student, no matter how innumerate, is capable of passing a mathematics examination. All that is required is a commitment to study and a willingness to suspend any prejudices about the subject gained at school. The fact that you have bothered to buy this book at all suggests that you are prepared to do both.

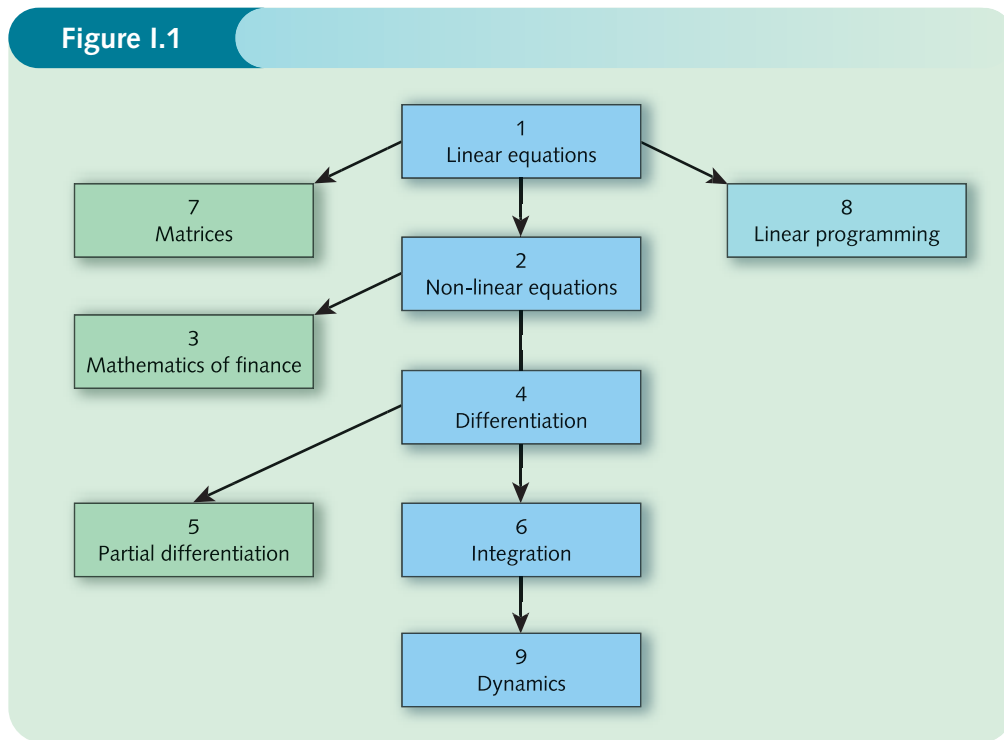
To help you get the most out of this book, let me compare the working practices of economics and engineering students. The former rarely read individual books in any great depth. They tend to visit college libraries (usually several days after an essay was due to be handed in) and to skim through a large number of books picking out the relevant information. Indeed, the ability to read selectively and

to compare various sources of information is an important skill that all arts and social science students must acquire. Engineering students, on the other hand, are more likely to read just a few books in any one year. They read each of these from cover to cover and attempt virtually every problem *en route*. Even though you are most definitely not an engineer, it is the engineering approach that you need to adopt while studying mathematics. There are several reasons for this. Firstly, a mathematics book can never be described, even by its most ardent admirers, as a good bedtime read. It can take an hour or two of concentrated effort to understand just a few pages of a mathematics text. You are therefore recommended to work through this book systematically in short bursts rather than to attempt to read whole chapters. Each section is designed to take between one and two hours to complete and this is quite sufficient for a single session. Secondly, mathematics is a hierarchical subject in which one topic follows on from the next. A construction firm building an office block is hardly likely to erect the fiftieth storey without making sure that the intermediate floors and foundations are securely in place. Likewise, you cannot 'dip' into the middle of a mathematics book and expect to follow it unless you have satisfied the prerequisites for that topic. Finally, you actually need to do mathematics yourself before you can understand it. No matter how wonderful your lecturer is, and no matter how many problems are discussed in class, it is only by solving problems yourself that you are ever going to become confident in using and applying mathematical techniques. For this reason, several problems are interspersed within the text and you are encouraged to tackle these as you go along. You will require writing paper, graph paper, pens and a calculator for this. There is no need to buy an expensive calculator unless you are feeling particularly wealthy at the moment. A bottom-of-the-range **scientific** calculator should be good enough. Detailed solutions are provided at the end of this book so that you can check your answers. However, please avoid the temptation to look at them until you have made an honest attempt at each one. Remember that in the future you may well have to sit down in an uncomfortable chair, in front of a blank sheet of paper, and be expected to produce solutions to examination questions of a similar type.

At the end of each section there are some further practice problems to try. You may prefer not to bother with these and to work through them later as part of your revision. Ironically, it is those students who really ought to try more problems who are most likely to miss them out. Human psychology is such that, if students do not at first succeed in solving problems, they are then deterred from trying additional problems. However, it is precisely these people who need more practice.

The chapter dependence is shown in Figure I.1. If you have studied some advanced mathematics before then you will discover that parts of Chapters 1, 2 and 4 are familiar. However, you may find that the sections on economics applications contain new material. You are best advised to test yourself by attempting a selection of problems in each section to see if you need to read through it as part of a refresher course. Economics students in a desperate hurry to experience the delights of calculus can miss out Chapter 3 without any loss of continuity and move straight on to Chapter 4. The mathematics of finance is probably more relevant to business and accountancy students, although you can always read it later if it is part of your economics syllabus.

Figure I.1



I hope that this book helps you to succeed in your mathematics course. You never know, you might even enjoy it. Remember to wear your engineer's hat while reading the book. I have done my best to make the material as accessible as possible. The rest is up to you!

## Getting started with Excel

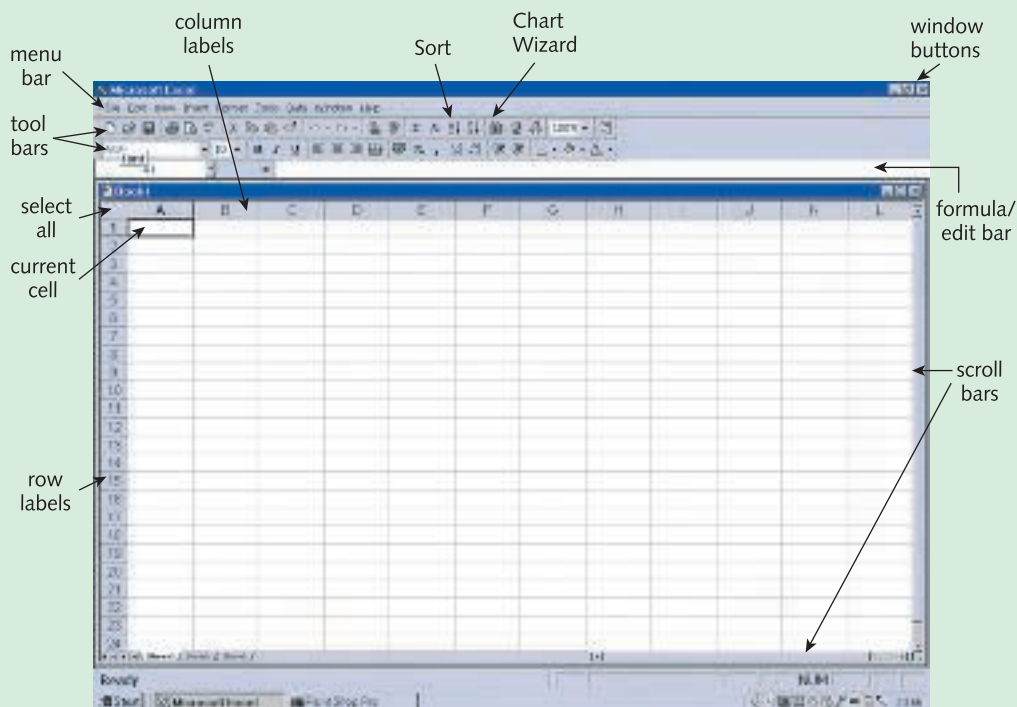
Excel is the Microsoft® spreadsheet package that we shall be using in some of our worked examples. If you are already familiar with this product, you may be able to skip some, or all, of this introductory section.

A spreadsheet is simply an array of boxes, or cells, into which tables of data can be inserted. This can consist of normal text, numerical data or a formula, which instructs the spreadsheet package to perform a calculation. The joy about getting the spreadsheet to perform the calculation is that it not only saves us some effort, but also detects any subsequent changes we make to the table, and recalculates its values automatically without waiting to be asked.

To get the most out of this section, it is advisable to work through it on your own computer, as there is no substitute for having a go. When you enter the Excel package, either by double-clicking the icon on your desktop, or by selecting it from the list of programs, a blank worksheet will be displayed, as shown in Figure I.2 (overleaf).

Each cell is identified uniquely by its column and row label. The current cell is where the cursor is positioned. In Figure I.2, the cursor is in the top left-hand corner: the cell is highlighted, and it can be identified as cell A1.

Figure I.2



### Example

### EXCEL

A shop audits its toy department to see how much profit it makes from sales of its five best-selling lines. Table I.1 shows the wholesale price (which is the cost to the shop of buying the toy from the manufacturer), the retail price (which is the price that customers pay for each toy), and sales (which is the total number of toys of each type that are sold during the year).

- Enter the information in this table into a blank spreadsheet, with the title, Annual Profit, in the first row.
- In a fifth column, calculate the annual profit generated by each toy and hence find the total profit made from all five toys.
- Format and print the completed spreadsheet.

Table I.1

| Item             | Wholesale price (\$) | Retail price (\$) | Sales |
|------------------|----------------------|-------------------|-------|
| Badminton racket | 28                   | 58                | 236   |
| Doll             | 36                   | 85                | 785   |
| Silly Putty      | 1                    | 2                 | 472   |
| Paddling pool    | 56                   | 220               | 208   |
| Building bricks  | 8                    | 26                | 582   |

## Solution

### (a) Entering the data

You can move between the different cells on the spreadsheet using the tab keys or arrow keys, or by positioning the cursor in the required cell and clicking the left mouse button. Have a go at this on your blank sheet to get the feel of it before we begin to enter the data.

To give the spreadsheet a title, we position the cursor in cell A1, and type Annual Profit. Don't worry that the text has run into the next cell. This does not matter, as we are not going to put anything more in this row.

Leaving the next row blank, we type in the column headings for the spreadsheet in row 3. To do this, we position the cursor in cell A3 and type Item; we then move the cursor to cell B3, and type Wholesale price (\$). At this stage, the spreadsheet looks like:

|   | A             | B                    | C |
|---|---------------|----------------------|---|
| 1 | Annual Profit |                      |   |
| 2 |               |                      |   |
| 3 | Item          | Wholesale price (\$) |   |
| 4 |               |                      |   |

This text has also run into the next cell. Although it looks as if we are positioned in C3 now, we are actually still in B3, as shown by the highlighting. The cursor can be positioned in cell C3 by using the tab, or right arrow key to give:

|   | A             | B                    | C |
|---|---------------|----------------------|---|
| 1 | Annual Profit |                      |   |
| 2 |               |                      |   |
| 3 | Item          | Wholesale price (\$) |   |
| 4 |               |                      |   |

Notice that the next cell is highlighted, even though it still contains our previous typing. We can ignore this, and enter Retail price (\$). As soon as you start entering this, the previous typing disappears. It is actually still there, but hidden from view as its own cell is not large enough to show all of its contents:

|   | A             | B                    | C                 | D |
|---|---------------|----------------------|-------------------|---|
| 1 | Annual Profit |                      |                   |   |
| 2 |               |                      |                   |   |
| 3 | Item          | Wholesale price (\$) | Retail price (\$) |   |
| 4 |               |                      |                   |   |

There is no need to worry about the hidden typing. We will sort this out when we format our spreadsheet in part (c). Finally, we position the cursor in cell D3 and type in the heading Sales.

We can now enter the names of the five items in cells A4 to A8, together with the prices and sales in columns B, C and D to create the spreadsheet:

|    | A             | B                    | C                 | D     | E |
|----|---------------|----------------------|-------------------|-------|---|
| 1  | Annual Profit |                      |                   |       |   |
| 2  |               |                      |                   |       |   |
| 3  | Item          | Wholesale price (\$) | Retail price (\$) | Sales |   |
| 4  | Dubliner      | 20                   | 50                | 200   |   |
| 5  | Bell          | 38                   | 85                | 765   |   |
| 6  | City Duty     | 1                    | 7                 | 477   |   |
| 7  | Paddington    | 38                   | 220               | 200   |   |
| 8  | Building      | 0                    | 23,500            |       |   |
| 9  |               |                      |                   |       |   |
| 10 |               |                      |                   |       |   |





If you subsequently return to modify the contents of any particular cell, you will find that when you start typing, the original contents of the cell are deleted, and replaced. If you simply want to amend, rather than replace the text, highlight the relevant cell, and then position the cursor at the required position in the original text, *which is displayed on the edit bar*. You can then edit the text as normal.

### (b) Calculating profit

In order to create a fifth column containing the profits, we first type the heading Profit in cell E3. Excel is capable of performing calculations and entering the results in particular cells. This is achieved by typing mathematical formulae into these cells. In this case, we need to enter an appropriate formula for profit in cells E4 to E8.

The profit made on each item is the difference between the wholesale price and retail price. For example, the shop buys a badminton racket from the manufacturer for \$28 and sells it to the customers at \$58. The profit made on the sale of a single racket is therefore

$$58 - 28 = 30$$

During the year the shop sells 236 badminton rackets, so the annual profit is

$$30 \times 236 = 7080$$

In other words, the profit on the sale of badminton rackets is worked out from

$$(58 - 28) \times 236$$

Looking carefully at the spreadsheet, notice that the numbers 58, 28 and 236 are contained in cells C4, B4 and D4, respectively. Hence annual profit made from the sale of badminton rackets is given by the formula

$$(C4-B4)*D4$$

in Excel the  
multiplication  
sign is \*

We would like the result of this calculation to appear underneath the heading Profit, in column 5, so in cell E4 we type

$$=(C4-B4)*D4$$

in Excel always  
start a formula  
with =

If you move the cursor down to cell E5, you will notice that the formula has disappeared, and the answer, 7080, has appeared in its place. To get back to the formula, click on cell E4, and the formula is displayed in the formula bar, where it can be edited if necessary.

We would like a similar formula to be entered into every cell in column E, to work out the profit generated by each type of toy. To avoid having to re-enter a similar formula for every cell, it is possible to replicate the one we just put into E4 down the whole column. The spreadsheet will automatically change the cell identities as we go.

To do this, position the cursor in E4, and move the mouse very carefully towards the bottom right-hand corner of the *cell* until the cursor changes from a  $\boxplus$  to a  $\boxtimes$ . Hold down the left mouse button and drag the cell down the column to E8. When the mouse button is released, the values of the profit will appear in the relevant cells.

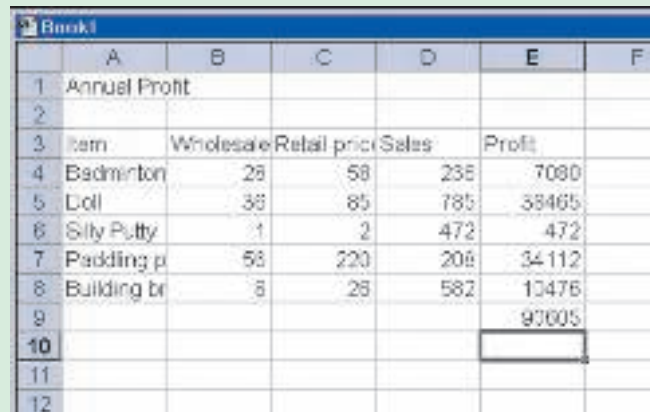
To put the total profit into cell E9, we need to sum up cells E4 to E8. This can be done by typing

$$=SUM(E4:E8)$$

into E9. Pressing the Enter key will then display the answer, 90 605, in this position.

The spreadsheet is displayed in Figure I.3.

Figure I.3



|    | A             | B         | C            | D     | E      | F |
|----|---------------|-----------|--------------|-------|--------|---|
| 1  | Annual Profit |           |              |       |        |   |
| 2  |               |           |              |       |        |   |
| 3  | Item          | Wholesale | Retail price | Sales | Profit |   |
| 4  | Badminton     | 28        | 58           | 238   | 7080   |   |
| 5  | Doll          | 36        | 85           | 785   | 38465  |   |
| 6  | Silly Putty   | 1         | 2            | 472   | 472    |   |
| 7  | Padding p     | 58        | 220          | 206   | 34112  |   |
| 8  | Building br   | 8         | 28           | 582   | 10476  |   |
| 9  |               |           |              |       | 97605  |   |
| 10 |               |           |              |       |        |   |
| 11 |               |           |              |       |        |   |
| 12 |               |           |              |       |        |   |

### (c) Formatting and printing the spreadsheet

Before we can print the spreadsheet we need to format it, to make it look more attractive to read. In particular, we must alter the column widths to reveal the partially hidden headings. If necessary, we can also insert or delete rows and columns. Perhaps the most useful function is the Undo, which reverses the previous action. If you do something wrong and want to go back a stage, simply click on the ↶ button, which is located towards the middle of the toolbar.

Here is a list of four useful activities that we can easily perform to tidy up the spreadsheet.

#### Adjusting the column widths to fit the data

Excel can automatically adjust the width of each column to reveal the hidden typing. You can either select an individual column by clicking on its label, or select all the columns at once by clicking the Select All button in the top left-hand corner (see Figure I.2 earlier). From the menu bar we then select **Format: Column: Autofit Selection**. The text that was obscured, because it was too long to fit into the cells, will now be displayed.

#### Shading and borders

Although the spreadsheet appears to have gridlines around each of the cells, these will not appear on the final printout unless we explicitly instruct Excel to do so. This can be done by highlighting the cells A3 to E8 by first clicking on cell A3, and then with the left mouse button held down, dragging the cursor across the table until all the cells are highlighted. We then release the mouse button, and select **Format: Cells** via the menu bar. Click on the **Border** tab, choose a style, and click on the boxes so that each cell is surrounded on all four sides by gridlines.

#### Sorting data into alphabetical order

It is sometimes desirable to list items in alphabetical order. To do this, highlight cells A4 to E8, by clicking and dragging, and then click the A → Z button on the toolbar.

#### Printing the spreadsheet

Before printing a spreadsheet, it is a good idea to select **File: Print Preview** from the menu bar to give you some idea of what it will look like. To change the orientation of the paper, select **File: Page Setup**. Additional



Figure I.4

| Item             | Wholesale price (\$) | Retail price (\$) | Sales | Profit |
|------------------|----------------------|-------------------|-------|--------|
| Badminton racket | 28                   | 58                | 238   | 7080   |
| Building bricks  | 8                    | 26                | 582   | 10478  |
| Doll             | 36                   | 85                | 785   | 36465  |
| Padding pool     | 56                   | 220               | 208   | 34112  |
| Silly Putty      | 1                    | 2                 | 472   | 472    |
| Total:           |                      |                   | 90605 |        |

features can be introduced such as headers, footers, column headings repeated at the top of every page, and so on. You might like to experiment with some of these to discover their effect. When you are happy, either click on the Print button, or select **File: Print** from the menu bar.

The final printout is shown in Figure I.4. As you can see, we have chosen to type in the text **Total:** in cell D9 and have also put gridlines around cells D9 and E9, for clarity.

### Practice Problem

**EXCEL**

- 1 An economics examination paper is in two sections. Section A is multiple choice and marked out of 40, whereas Section B consists of essay questions and is marked out of 60. Table I.2 shows the marks awarded in each section to six candidates.

Table I.2

| Candidate | Section A mark | Section B mark |
|-----------|----------------|----------------|
| Fofaria   | 20             | 17             |
| Bull      | 38             | 12             |
| Eoin      | 34             | 38             |
| Arefin    | 40             | 52             |
| Cantor    | 29             | 34             |
| Devaux    | 30             | 49             |

- (a) Enter the information in this table into a blank spreadsheet, with the title, Economics Examination Marks, in the first row.
- (b) In a fourth column, calculate the total mark awarded to each candidate.
- (c) Use Excel to calculate the average examination mark of these six candidates and give it an appropriate heading.
- (d) Format and print the spreadsheet, putting the names of the candidates in alphabetical order.
- (e) The second candidate, Bull, asks for a re-mark. Although the Section A mark is correct, the Section B mark is raised to 42. Produce a new spreadsheet based on the correct results.

## Getting started with Maple

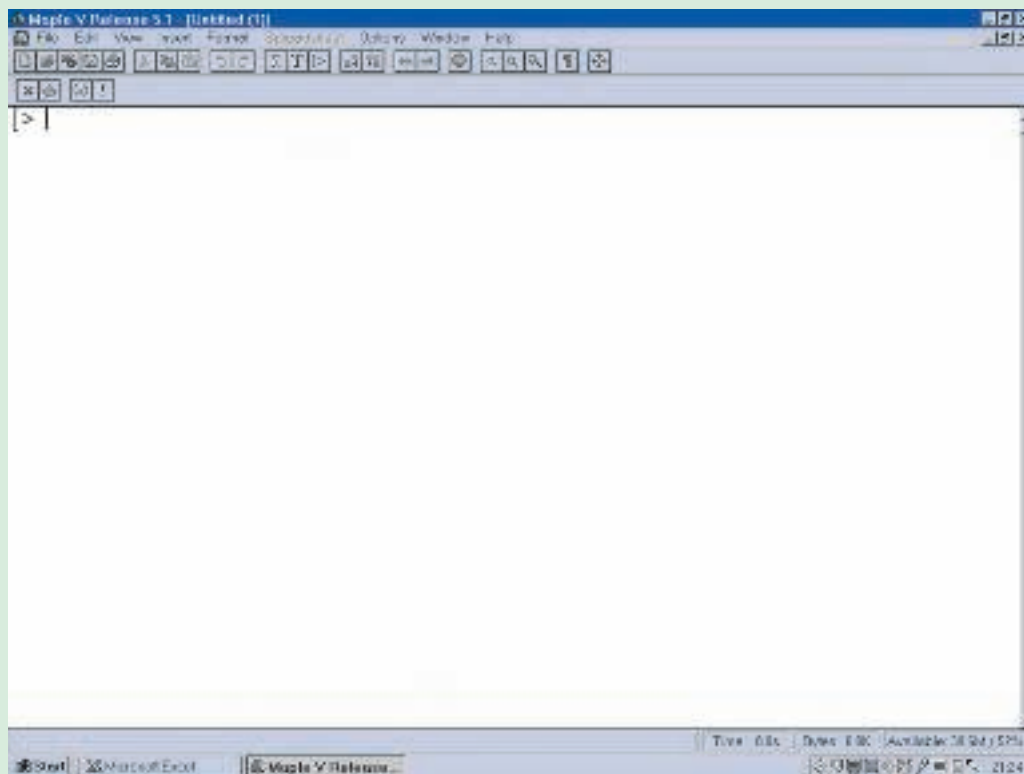
The second computer package that will be used in this book is Maple. This is a symbolic algebra system. It not only performs numerical calculations but also manipulates mathematical symbols. In effect, it obligingly does the mathematics for you. There are other similar packages available, such as Matlab, Derive and Mathcad, and most of the Maple examples and exercises given in this book can be tackled just as easily using these packages instead. This is not the place to show you the full power of Maple, but hopefully the examples given in this book will give you a flavour of what can be achieved, and why it is such a valuable tool in mathematical modelling.

It is not possible in this introductory section to use Maple to solve realistic problems because you need to learn some mathematics first. However, we will show you how to use it as a calculator, and how to type in mathematical formulae correctly. Figure I.5 shows a typical worksheet which appears on the screen when you double-click on the Maple icon. If you ignore the toolbar at the top of the screen, you can think of it as a blank sheet of paper on which to do some mathematics. You type this after the '>' prompt and end each instruction with a semi-colon ';'. Pressing the Enter key will then make Maple perform your instruction and give you an answer. For example, if you want Maple to work out  $3 + 4 \times 2$  you type:

>3+4\*2;

notice the  
semi-colon

Figure I.5



After pressing the Enter key, the package will respond with the answer of 11. Try it now.

Notice that to get this answer, Maple must have performed the multiplication first (to get 8) before adding on the 3. This is because, like the rest of the mathematical world, Maple follows the BIDMAS convention:

|         |   |                  |                            |
|---------|---|------------------|----------------------------|
|         | B | (Brackets first) |                            |
|         | I | (Indices second) |                            |
| then    | D | (Division)       | } (a tie for third place)  |
| and     | M | (Multiplication) |                            |
| finally | A | (Addition)       | } (a tie for fourth place) |
| and     | S | (Subtraction)    |                            |

Since multiplication has a higher priority than addition, Maple works out  $4 \times 2$  first. If you really want to work out  $3 + 4$  before multiplying by 2, you put in brackets:

`> (3+4) * 2 ;`

don't forget to put the semi-colon at the end

which gives 14. Notice that in Maple (like Excel), it is necessary to use `*` rather than  $\times$  as the multiplication symbol. Similarly, Maple and Excel both use `/` (instead of  $\div$ ) for division, and `^` as the instruction to 'raise to the power of'. So, to work out  $\frac{70}{7} + 5^2$  you type:

`> 70/7+5^2 ;`

which gives 35. Again, using BIDMAS, since indices have a higher priority than addition,  $5^2$  has been evaluated first as 25, before adding on  $70/7 = 10$ , to give a final answer of 35.

## Practice Problem

## MAPLE

2 Use Maple to work out each of the following:

(a)  $12 + 18 \div 9$

(b)  $3^3 - 4^2$

(c)  $(7 + 3) \div 2$

Suppose now that you wish to work out all of the following sums:

$$3 \times 5^2 - 2 \times 5$$

$$3 \times 6^2 - 2 \times 6$$

$$3 \times 7^2 - 2 \times 7$$

$$3 \times 6.4^2 - 2 \times 6.4$$

$$3 \times 92.5^2 - 2 \times 92.5$$

You could, of course, just type all five calculations, one after the other to get the answers. However, there is a common pattern. They each take the form

$$3x^2 - 2x$$

for various values of  $x$ , and it makes sense to exploit this fact. As a first step, we shall give this expression a name. We could call it Fred or Wilma, but in practice, we prefer to give it a name that relates to the context in which it arises. A mathematical expression that contains a square term like this is called a quadratic so let us name this particular one quad1. To do this, type:

```
>quad1:=3*x^2-2*x;
```

The symbol '=' tells Maple that you wish to *define* quad1 to be  $3x^2 - 2x$ .

To work out  $3 \times 5^2 - 2 \times 5$  we substitute  $x = 5$  into this expression: that is, we replace the symbol  $x$  by the number 5. In Maple, this is achieved by typing

```
>subs(x=5,quad1);
```

Maple responds with the answer 65. To perform the other four calculations, all we need do is to edit the Maple instruction, change the 5 to a new value, move the cursor to the right of the semi-colon, and press the Enter key. You might like to try this for yourself.

Hopefully, this brief introduction has given you some idea of how to use Maple to perform simple calculations. However, you may be wondering what all the fuss is about. Surely we could have performed these calculations just as easily on an ordinary calculator? Well, the honest answer is probably yes. The real advantage of using Maple is as a tool for solving complex mathematical problems. We shall meet some examples in the second half of this book. The following problem gives a brief glimpse at the sort of things that it can do.

### Practice Problem

### MAPLE

**3** Type in the following after the '>' prompt. Before doing so, you might like to guess what each instruction is likely to do.

- (a) `solve(2*x-8=0,x);`
- (b) `plot(2*x-8,x=0..10);`
- (c) `expand((x+2)^2);`
- (d) `simplify(2*x+6+5*x-2);`
- (e) `plot3d(x^3-3*x+x*y,x=-2..2,y=-2..2);`

### Key Terms

The key terms in each section are shown in *colour* in the text when they are first used. Their definitions are shown at the end of each section in a coloured box. In addition, all key terms are repeated in the glossary at the end of the book.





# chapter 1

## Linear Equations

The main aim of this chapter is to introduce the mathematics of linear equations. This is an obvious first choice in an introductory text, since it is an easy topic which has many applications. There are six sections, which are intended to be read in the order that they appear.

Sections 1.1, 1.2, 1.4 and 1.5 are devoted to mathematical methods. They serve to revise the rules of arithmetic and algebra, which you probably met at school but may have forgotten. In particular, the properties of negative numbers and fractions are considered. A reminder is given on how to multiply out brackets and how to manipulate mathematical expressions. You are also shown how to solve simultaneous linear equations. Systems of two equations in two unknowns can be solved using graphs, which are described in Section 1.1. However, the preferred method uses elimination, which is considered in Section 1.2. This algebraic approach has the advantage that it always gives an exact solution and it extends readily to larger systems of equations.

The remaining two sections are reserved for applications in microeconomics and macroeconomics. You may be pleasantly surprised by how much economic theory you can analyse using just the basic mathematical tools considered here. Section 1.3 introduces the fundamental concept of an economic function and describes how to calculate equilibrium prices and quantities in supply and demand theory. Section 1.6 deals with national income determination in simple macroeconomic models.

The first five sections underpin the rest of the book and are essential reading. The final section is not quite as important and can be omitted if desired.



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