

Mechanics of Fluids

Eighth edition

Bernard Massey

*Reader Emeritus in Mechanical Engineering
University College, London*

Revised by

John Ward-Smith

*Formerly Senior Lecturer in Mechanical Engineering
Brunel University*



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Preface to the eighth edition

In this eighth edition, the aim has been to build on the broad ethos established in the first edition and maintained throughout all subsequent editions. The purpose of the book is to present the basic principles of fluid mechanics and to illustrate them by application to a variety of problems in different branches of engineering. The book contains material appropriate to an honours degree course in mechanical engineering, and there is also much that is relevant to undergraduate courses in aeronautical, civil and chemical engineering.

It is a book for engineers rather than mathematicians. Particular emphasis is laid on explaining the physics underlying aspects of fluid flow. Whilst mathematics has an important part to play in this book, specialized mathematical techniques are deliberately avoided. Experience shows that fluid mechanics is one of the more difficult and challenging subjects studied by the undergraduate engineer. With this in mind the presentation has been made as user-friendly as possible. Students are introduced to the subject in a systematic way, the text moving from the simple to the complex, from the familiar to the unfamiliar.

Two changes relating to the use of SI units appear in this eighth edition and are worthy of comment. First, in recognition of modern developments, the representation of derived SI units is different from that of previous editions. Until recently, two forms of unit symbol were in common use and both are still accepted within SI. However, in recent years, in the interests of clarity, there has been a strong movement in favour of a third form. The *half-high dot* (also known as the *middle dot*) is now widely used in scientific work in the construction of derived units. This eighth edition has standardized on the use of the half-high dot to express SI units. The second change is as follows: for the first time SI units are used throughout. In particular, in dealing with rotational motion, priority is given to the use of the SI unit of angular velocity ($\text{rad} \cdot \text{s}^{-1}$ supplanting rev/s).

The broad structure of the book remains the same, with thirteen chapters. However, in updating the previous edition, many small revisions and a number of more significant changes have been made. New material has been introduced, some text has been recast, certain sections of text have been moved between chapters, and some material contained in earlier editions has been omitted. Amongst the principal changes, Chapter 1

has been substantially revised and expanded. Its purpose is to provide a broad introduction to fluid mechanics, as a foundation for the more detailed discussion of specific topics contained in the remaining chapters. Fluid properties, units and dimensions, terminology, the different types of fluid flow of interest to engineers, and the roles of experimentation and mathematical theory are all touched on here. The treatment of dimensional analysis (Chapter 5) has been revised. A number of topics are covered for the first time, including the losses arising from the flow through nozzles, orifice meters, gauzes and screens (Chapter 7). The concept of the friction velocity has been brought in to Chapter 8, and the theory of functions of a complex variable and its application to inviscid flows is set down in Chapter 9. A discussion of the physics of tsunamis has been added to Chapter 10. In Chapter 11, changes include the addition of material on the mass flow parameters in compressible flow. Finally, in Chapter 13, the treatment of dimensionless groups has been changed to reflect the use of SI units, and new material on the selection of pumps and fans has been introduced.

Footnotes, references and suggestions for further reading, which were included in earlier editions, have been removed. The availability of information retrieval systems and search engines on the internet has enabled the above changes to be introduced in this edition. It is important that students become proficient at using these new resources. Searching by keyword, author or subject index, the student has access to a vast fund of knowledge to supplement the contents of this book, which is intended to be essentially self-contained.

It remains to thank those, including reviewers and readers of previous editions, whose suggestions have helped shape this book.

February 2005

Fundamental concepts

1

The aim of Chapter 1 is to provide a broad introduction to fluid mechanics, as a foundation for the more detailed discussion of specific topics contained in Chapters 2–13. We start by considering the characteristics of liquids and gases, and what it is that distinguishes them from solids. The ability to measure and quantify fluid and flow properties is of fundamental importance in engineering, and so at an early stage the related topics of units and dimensions are introduced. We move on to consider the properties of fluids, such as density, pressure, compressibility and viscosity. This is followed by a discussion of the terminology used to describe different flow patterns and types of fluid motion of interest to engineers. The chapter concludes by briefly reviewing the roles of experimentation and mathematical theory in the study of fluid mechanics.

1.1 THE CHARACTERISTICS OF FLUIDS

A fluid is defined as a substance that deforms continuously whilst acted upon by any force tangential to the area on which it acts. Such a force is termed a *shear force*, and the ratio of the shear force to the area on which it acts is known as the *shear stress*. Hence when a fluid is at rest neither shear forces nor shear stresses exist in it. A solid, on the other hand, can resist a shear force while at rest. In a solid, the shear force may cause some initial displacement of one layer over another, but the material does not continue to move indefinitely and a position of stable equilibrium is reached. In a fluid, however, shear forces are possible only while relative movement between layers is taking place. A fluid is further distinguished from a solid in that a given amount of it owes its shape at any time to that of the vessel containing it, or to forces that in some way restrain its movement.

The distinction between solids and fluids is usually clear, but there are some substances not easily classified. Some fluids, for example, do not flow easily: thick tar or pitch may at times appear to behave like a solid. A block of such a substance may be placed on the ground, and, although its flow would take place very slowly, over a period of time – perhaps several days – it would spread over the ground by the action of gravity. On

2 Fundamental concepts

the other hand, certain solids may be made to ‘flow’ when a sufficiently large force is applied; these are known as *plastic solids*. Nevertheless, these examples are rather exceptional and outside the scope of mainstream fluid mechanics.

The essential difference between solids and fluids remains. Any fluid, no matter how *thick* or *viscous* it is, flows under the action of a net shear force. A solid, however, no matter how plastic it is, does not flow unless the net shear force on it exceeds a certain value. For forces less than this value the layers of the solid move over one another only by a certain amount. The more the layers are displaced from their original relative positions, the greater are the internal forces within the material that resist the displacement. Thus, if a steady external force is applied, a state will be reached in which the internal forces resisting the movement of one layer over another come into balance with the external applied force and so no further movement occurs. If the applied force is then removed, the resisting forces within the material will tend to restore the solid body to its original shape.

In a fluid, however, the forces opposing the movement of one layer over another exist only while the movement is taking place, and so static equilibrium between applied force and resistance to shear never occurs. Deformation of the fluid takes place continuously so long as a shear force is applied. But if this applied force is removed the shearing movement subsides and, as there are then no forces tending to return the particles of fluid to their original relative positions, the fluid keeps its *new* shape.

Liquid

Fluids may be sub-divided into liquids and gases. A fixed amount of a liquid has a definite volume which varies only slightly with temperature and pressure. If the capacity of the containing vessel is greater than this definite volume, the liquid occupies only part of the container, and it forms an interface separating it from its own vapour, the atmosphere or any other gas present.

Gas

A fixed amount of a gas, by itself in a closed container, will always expand until its volume equals that of the container. Only then can it be in equilibrium. In the analysis of the behaviour of fluids an important difference between liquids and gases is that, whereas under ordinary conditions liquids are so difficult to compress that they may for most purposes be regarded as incompressible, gases may be compressed much more readily. Where conditions are such that an amount of gas undergoes a negligible change of volume, its behaviour is similar to that of a liquid and it may then be regarded as incompressible. If, however, the change in volume is not negligible, the compressibility of the gas must be taken into account in examining its behaviour.

A second important difference between liquids and gases is that liquids have much greater densities than gases. As a consequence, when considering forces and pressures that occur in fluid mechanics, the weight of a liquid has an important role to play. Conversely, effects due to weight can usually be ignored when gases are considered.

1.1.1 Molecular structure

The different characteristics of solids, liquids and gases result from differences in their molecular structure. All substances consist of vast numbers of molecules separated by empty space. The molecules have an attraction for one another, but when the distance between them becomes very small (of the order of the diameter of a molecule) there is a force of repulsion between them which prevents them all gathering together as a solid lump.

The molecules are in continual movement, and when two molecules come very close to one another the force of repulsion pushes them vigorously apart, just as though they had collided like two billiard balls. In solids and liquids the molecules are much closer together than in a gas. A given volume of a solid or a liquid therefore contains a much larger number of molecules than an equal volume of a gas, so solids and liquids have a greater density (i.e. mass divided by volume).

In a solid, the movement of individual molecules is slight – just a vibration of small amplitude – and they do not readily move relative to one another. In a liquid the movement of the molecules is greater, but they continually attract and repel one another so that they move in curved, wavy paths rather than in straight lines. The force of attraction between the molecules is sufficient to keep the liquid together in a definite volume although, because the molecules can move past one another, the substance is not rigid. In a gas the molecular movement is very much greater; the number of molecules in a given space is much less, and so any molecule travels a much greater distance before meeting another. The forces of attraction between molecules – being inversely proportional to the square of the distance between them – are, in general, negligible and so molecules are free to travel away from one another until they are stopped by a solid or liquid boundary.

The activity of the molecules increases as the temperature of the substance is raised. Indeed, the temperature of a substance may be regarded as a measure of the average kinetic energy of the molecules.

When an external force is applied to a substance the molecules tend to move relative to one another. A solid may be deformed to some extent as the molecules change position, but the strong forces between molecules remain, and they bring the solid back to its original shape when the external force is removed. Only when the external force is very large is one molecule wrenched away from its neighbours; removal of the external force does not then result in a return to the original shape, and the substance is said to have been deformed beyond its elastic limit.

In a liquid, although the forces of attraction between molecules cause it to hold together, the molecules can move past one another and find new neighbours. Thus a force applied to an unconfined liquid causes the molecules to slip past one another until the force is removed.

If a liquid is in a confined space and is compressed it exhibits elastic properties like a solid in compression. Because of the close spacing of the molecules, however, the resistance to compression is great. A gas, on the other hand, with its molecules much farther apart, offers much less resistance to compression.

1.1.2 The continuum

An absolutely complete analysis of the behaviour of a fluid would have to account for the action of each individual molecule. In most engineering applications, however, interest centres on the *average* conditions of velocity, pressure, temperature, density and so on. Therefore, instead of the actual conglomeration of separate molecules, we regard the fluid as a *continuum*, that is a continuous distribution of matter with no empty space. This assumption is normally justifiable because the number of molecules involved in the situation is so vast and the distances between them are so small. The assumption fails, of course, when these conditions are not satisfied as, for example, in a gas at extremely low pressure. The average distance between molecules may then be appreciable in comparison with the smallest significant length in the fluid boundaries. However, as this situation is well outside the range of normal engineering work, we shall in this book regard a fluid as a continuum. Although it is often necessary to postulate a small element or particle of fluid, this is supposed large enough to contain very many molecules.

The properties of a fluid, although molecular in origin, may be adequately accounted for in their overall effect by ascribing to the continuum such attributes as temperature, pressure, viscosity and so on. Quantities such as velocity, acceleration and the properties of the fluid are assumed to vary continuously (or remain constant) from one point to another in the fluid.

The new field of nanotechnology is concerned with the design and fabrication of products at the molecular level, but this topic is outside the scope of this text.

1.1.3 Mechanics of fluids

The *mechanics of fluids* is the field of study in which the fundamental principles of general mechanics are applied to liquids and gases. These principles are those of the conservation of matter, the conservation of energy and Newton's laws of motion. In extending the study to compressible fluids, we also need to consider the laws of thermodynamics. By the use of these principles, we are not only able to explain observed phenomena, but also to predict the behaviour of fluids under specified conditions. The study of the mechanics of fluids can be further sub-divided. For fluids at rest the study is known as *fluid statics*, whereas if the fluid is in motion, the study is called *fluid dynamics*.

1.2 NOTATION, DIMENSIONS, UNITS AND RELATED MATTERS

Calculations are an important part of engineering fluid mechanics. Fluid and flow properties need to be quantified. The overall designs of aircraft and dams, just to take two examples, depend on many calculations, and if errors are made at any stage then human lives are put at risk. It is vital,

therefore, to have in place systems of measurement and calculation which are consistent, straightforward to use, minimize the risk of introducing errors, and allow checks to be made. These are the sorts of issues that we consider in detail here.

1.2.1 Definitions, conventions and rules

In the physical sciences, the word *quantity* is used to identify any physical attribute capable of representation by measurement. For example, mass, weight, volume, distance, time and velocity are all quantities, according to the sense in which the word is used in the scientific world. The *value* of a quantity is defined as the magnitude of the quantity expressed as the product of a number and a unit. The number multiplying the unit is the *numerical value* of the quantity expressed in that unit. (The *numerical value* is sometimes referred to as the *numeric*.) A *unit* is no more than a particular way of attaching a numerical value to the quantity, and it is part of a wider scene involving a *system of units*. Units within a system of units are of two kinds. First, there are the *base units* (or *primary units*), which are mutually independent. Taken together, the base units define the *system of units*. Then there are the *derived units* (or *secondary units*) which can be determined from the definitions of the base units.

Each quantity has a *quantity name*, which is spelt out in full, or it can be represented by a *quantity symbol*. Similarly, each unit has a *unit name*, which is spelt out in full, or it can be abbreviated and represented by a *unit symbol*. The use of symbols saves much space, particularly when setting down equations. Quantity symbols and unit symbols are mathematical entities and, since they are not like ordinary words or abbreviations, they have their own sets of rules. To avoid confusion, symbols for quantities and units are represented differently. Symbols for quantities are shown in *italic* type using letters from the Roman or Greek alphabets. Examples of quantity symbols are F , which is used to represent force, m mass, and so on. The definitions of the quantity symbols used throughout this book are given in Appendix 4. Symbols for units are not italicized, and are shown in Roman type. Subscripts or superscripts follow the same rules. Arabic numerals are used to express the numerical value of quantities.

In order to introduce some of the basic ideas relating to dimensions and units, consider the following example. Suppose that a velocity is reported as $30 \text{ m} \cdot \text{s}^{-1}$. In this statement, the number 30 is described as the *numeric* and $\text{m} \cdot \text{s}^{-1}$ are the *units* of measurement. The notation $\text{m} \cdot \text{s}^{-1}$ is an abbreviated form of the ratio metre divided by second. There are 1000 m in 1 km, and 3600 s in 1 h. Hence, a velocity of $30 \text{ m} \cdot \text{s}^{-1}$ is equivalent to $108 \text{ km} \cdot \text{h}^{-1}$. In the latter case, the *numeric* is 108 and the *units* are $\text{km} \cdot \text{h}^{-1}$. Thus, for defined units, the numeric is a measure of the *magnitude* of the velocity. The magnitude of a quantity is seen to depend on the units in which it is expressed.

Consider the variables: distance, depth, height, width, thickness. These variables have different meanings, but they all have one feature in

common – they have the *dimensions* of length. They can all be measured in the same units, for example metres. From these introductory considerations, we can move on to deal with general principles relating to the use of dimensions and units in an engineering context. The *dimension* of a variable is a fundamental statement of the physical nature of that variable. Variables with particular physical characteristics in common have the same dimensions; variables with different physical qualities have different dimensions. Altogether, there are seven primary dimensions but, in engineering fluid mechanics, just four of the primary dimensions – mass, length, time and temperature – are required. A *unit* of measurement provides a means of quantifying a variable. Systems of units are essentially arbitrary, and rely upon agreement about the definition of the primary units. This book is based on the use of SI units.

1.2.2 Units of the Système International d’Unités (SI units)

This system of units is an internationally agreed version of the metric system; since it was established in 1960 it has experienced a process of fine-tuning and consolidation. It is now employed throughout most of the world and will no doubt eventually come into universal use. An extensive and up-to-date guide, which has influenced the treatment of SI units throughout this book, is: Barry N. Taylor (2004). *Guide for the Use of the International System of Units (SI)* (version 2.2). [Online] Available: <http://physics.nist.gov/Pubs/SP811/contents.html> [2004, August 28]. National Institute of Standards and Technology, Gaithersburg, MD.

The seven primary SI units, their names and symbols are given in Table 1.1. In engineering fluid mechanics, the four primary units are: kilogram, metre, second and kelvin. These may be expressed in abbreviated form. For example, kilogram is represented by kg, metre by m, second by s and kelvin by K.

From these base or primary units, all other units, known as *derived* or *secondary* units, are constructed (e.g. $\text{m} \cdot \text{s}^{-1}$ as a unit of velocity). Over the years, the way in which these derived units are written has changed. Until recently, two abbreviated forms of notation were in common use. For example, metre/second could be abbreviated to m/s or m s^{-1} where, in the second example, a space separates the m and s. In recent years, there

Table 1.1 Primary SI units

<i>Quantity</i>	<i>Unit</i>	<i>Symbol</i>
length	metre	m
mass	kilogram	kg
time	second	s
electric current	ampere	A
thermodynamic temperature	kelvin	K (formerly °K)
luminous intensity	candela	cd
amount of substance	mole	mol

has been a strong movement in favour of a third form of notation, which has the benefit of clarity, and the avoidance of ambiguity. The *half-high dot* (also known as the *middle dot*) is now widely used in scientific work in the construction of derived units. Using the half-high dot, metre/second is expressed as $\text{m} \cdot \text{s}^{-1}$. The style based on the half-high dot is used throughout this book to represent SI units. (Note that where reference is made in this book to units which are outside the SI, such as in the discussion of conversion factors, the half-high dot notation will not be applied to non-SI units. Hence, SI units can be readily distinguished from non-SI units.)

Certain secondary units, derived from combinations of the primary units, are given internationally agreed special names. Table 1.2 lists those used in this book. Some other special names have been proposed and may be adopted in the future.

Although strictly outside the SI, there are a number of units that are accepted for use with SI. These are set out in Table 1.3.

The SI possesses the special property of coherence. A system of units is said to be coherent with respect to a system of quantities and equations if the system of units satisfies the condition that the equations between numerical values have exactly the same form as the corresponding equations between the quantities. In such a coherent system only the number 1 ever occurs as a numerical factor in the expressions for the derived units in terms of the base units.

Table 1.2 Names of some derived units

<i>Quantity</i>	<i>Unit</i>	<i>Symbol</i>	<i>Equivalent combination of primary units</i>
force	Newton	N	$\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$
pressure and stress	pascal	Pa	$\text{N} \cdot \text{m}^{-2}$ ($\equiv \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$)
work, energy, quantity of heat	joule	J	$\text{N} \cdot \text{m}$ ($\equiv \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$)
power	watt	W	$\text{J} \cdot \text{s}^{-1}$ ($\equiv \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$)
frequency	hertz	Hz	s^{-1}
plane angle	radian	rad	
solid angle	steradian	sr	

Table 1.3 Units accepted for use with the SI

<i>Name</i>	<i>Quantity</i>	<i>Symbol</i>	<i>Value in SI units</i>
minute	time	min	1 min = 60 s
hour	time	h	1 h = 60 min = 3600 s
day	time	d	1 d = 24 h = 86 400 s
degree	plane angle	°	1° = $(\pi/180)$ rad
minute	plane angle	'	1' = $(1/60)^\circ = (\pi/10\,800)$ rad
second	plane angle	"	1" = $(1/60)'$ = $(\pi/648\,000)$ rad
litre	volume	L	1 L = 1 dm ³ = 10 ⁻³ m ³
metric ton or tonne	mass	t	1 t = 10 ³ kg

1.2.3 Prefixes

To avoid inconveniently large or small numbers, prefixes may be put in front of the unit names (see Table 1.4). Especially recommended are prefixes that refer to factors of 10^{3n} , where n is a positive or negative integer.

Care is needed in using these prefixes. The symbol for a prefix should always be written close to the symbol of the unit it qualifies, for example, kilometre (km), megawatt (MW), microsecond (μs). Only one prefix at a time may be applied to a unit; thus 10^{-6} kg is 1 milligram (mg), *not* 1 microkilogram.

The symbol ‘m’ stands both for the basic unit ‘metre’ and for the prefix ‘milli’, so care is needed in using it. The introduction of the half-high dot has eliminated the risk of certain ambiguities associated with earlier representations of derived units.

When a unit with a prefix is raised to a power, the exponent applies to the *whole multiple* and not just to the original unit. Thus 1 mm^2 means $1(\text{mm})^2 = (10^{-3} \text{ m})^2 = 10^{-6} \text{ m}^2$, and *not* $1 \text{ m}(\text{m}^2) = 10^{-3} \text{ m}^2$.

The symbols for units refer not only to the singular but also to the plural. For instance, the symbol for kilometres is km, not kms.

Capital or lower case (small) letters are used strictly in accordance with the definitions, no matter in what combination the letters may appear.

Table 1.4 Prefixes for multiples and submultiples of SI units

<i>Prefix</i>	<i>Symbol</i>	<i>Factor by which unit is multiplied</i>
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deca	da	10
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

1.2.4 Comments on some quantities and units

In everyday life, temperatures are conventionally expressed using the Celsius temperature scale (formerly known as Centigrade temperature scale). The symbol °C is used to express Celsius temperature. The Celsius temperature (symbol t) is related to the thermodynamic temperature (symbol T) by the equation

Temperature

$$t = T - T_0$$

where $T_0 = 273.15$ K by definition. For many purposes, 273.15 can be rounded off to 273 without significant loss of accuracy. The thermodynamic temperature T_0 is exactly 0.01 K below the triple-point of water.

Note that 1 newton is the net force required to give a body of mass 1 kg an acceleration of $1 \text{ m} \cdot \text{s}^{-2}$.

Force

The weight W and mass m of a body are related by

*Gravitational
acceleration*

$$W = mg$$

The quantity represented by the symbol g is variously described as the gravitational acceleration, the acceleration of gravity, weight per unit mass, the acceleration of free fall and other terms. Each term has its merits and weaknesses, which we shall not discuss in detail here. Suffice it to say that we shall use the first two terms. As an acceleration, the units of g are usually represented in the natural form $\text{m} \cdot \text{s}^{-2}$, but it is sometimes convenient to express them in the alternative form $\text{N} \cdot \text{kg}^{-1}$, a form which follows from the definition of the newton.

Note that 1 pascal is the pressure induced by a force of 1 N acting on an area of 1 m^2 . The pascal, Pa, is small for most purposes, and thus multiples are often used. The bar, equal to 10^5 Pa, has been in use for many years, but as it breaks the 10^{3n} convention it is not an SI unit.

Pressure and stress

In the measurement of fluids the name *litre* is commonly given to 10^{-3} m^3 . Both l and L are internationally accepted symbols for the litre. However, as the letter l is easily mistaken for 1 (one), the symbol L is now recommended and is used in this book.

Volume

The SI unit for plane angle is the radian. Consequently, angular velocity has the SI unit $\text{rad} \cdot \text{s}^{-1}$. Hence, as SI units are used throughout this text, angular velocity, denoted by the symbol ω , is specified with the units $\text{rad} \cdot \text{s}^{-1}$.

Angular velocity

Another measure of plane angle, the revolution, equal to 360° , is not part of the SI, nor is it a unit accepted for use with SI (unlike the units degree, minute and second, see Table 1.3). The revolution, here abbreviated to rev, is easy to measure. In consequence rotational speed is widely reported in industry in the units rev/s. (We avoid using the half-high dot to demonstrate that the unit is not part of the SI.) It would be unrealistic to ignore the popularity of this unit of measure and so, where appropriate, supplementary

information on rotational speed is provided in the units rev/s. To distinguish the two sets of units, we retain the symbol ω for use when the angular velocity is measured in $\text{rad} \cdot \text{s}^{-1}$, and use the symbol N when the units are rev/s. Thus N is related to ω by the expression $N = \omega/2\pi$.

1.2.5 Conversion factors

This book is based on the use of SI units. However, other systems of units are still in use; on occasions it is necessary to convert data into SI units from these other systems of units, and vice versa. This may be done by using *conversion factors* which relate the sizes of different units of the same kind.

As an example, consider the identity

$$1 \text{ inch} \equiv 25.4 \text{ mm}$$

(The use of three lines (\equiv), instead of the two lines of the usual equals sign, indicates not simply that one inch equals or is equivalent to 25.4 mm but that one inch *is* 25.4 mm. At all times and in all places one inch and 25.4 mm are precisely the same.) The identity may be rewritten as

$$1 \equiv \frac{25.4 \text{ mm}}{1 \text{ inch}}$$

and this ratio *equal to unity* is a conversion factor. Moreover, as the reciprocal of unity is also unity, any conversion factor may be used in reciprocal form when the desired result requires it.

This simple example illustrates how a measurement expressed in one set of units can be converted into another. The principle may be extended indefinitely. A number of conversion factors are set out in Appendix 1.

If magnitudes are expressed on scales with different zeros (e.g. the Fahrenheit and Celsius scales of temperature) then unity conversion factors may be used only for *differences* of the quantity, not for individual points on a scale. For instance, a temperature *difference* of $36^\circ\text{F} = 36^\circ\text{F} \times (1^\circ\text{C}/1.8^\circ\text{F}) = 20^\circ\text{C}$, but a temperature of 36°F corresponds to 2.22°C , not 20°C .

1.2.6 Orders of magnitude

There are circumstances where great precision is not required and just a general indication of magnitude is sufficient. In such cases we refer to the *order of magnitude* of a quantity. To give meaning to the term, consider the following statements concerning examples taken from everyday life: the thickness of the human hair is of the order 10^{-4} m; the length of the human thumb nail is of order 10^{-2} m; the height of a human is of order 1 m; the height of a typical two-storey house is of order 10 m; the cruise altitude of a subsonic civil aircraft is of order 10^4 m. These examples cover a range of 8 orders of magnitude. The height of a human is typically 4 orders of magnitude larger than the thickness of the human hair. The cruise altitude of an airliner exceeds the height of a human by 4 orders of magnitude. In this context, it is unimportant that the height of most humans

is nearer 2 m, rather than 1 m. Here we are simply saying that the height of a human is closer to 1 m rather than 10 m, the next nearest order of magnitude.

As an example of the usefulness of order of magnitude considerations, let us return to the concept of the continuum; we can explain why the continuum concept is valid for the analysis of practical problems of fluid mechanics. For most gases, the mean free path – that is the distance that on average a gas molecule travels before colliding with another molecule – is of the order of 10^{-7} m and the average distance between the centres of neighbouring molecules is about 10^{-9} m. In liquids, the average spacing of the molecules is of the order 10^{-10} m. In contrast, the diameter of a hot-wire anemometer (see Chapter 7), which is representative of the smallest lengths at the macroscopic level, is of the order 10^{-4} m. The molecular scale is seen to be several (3 or more) orders smaller than the macroscopic scale of concern in engineering.

Arguments based on a comparison of the order of magnitude of quantities are of immense importance in engineering. Where such considerations are relevant – for example, when analysing situations, events or processes – factors which have a minor influence can be disregarded, allowing attention to be focused on the factors which really matter. Consequently, the physics is easier to understand and mathematical equations describing the physics can be simplified.

1.2.7 Dimensional formulae

The notation for the four primary dimensions is as follows: mass [M], length [L], time [T] and temperature [Θ]. The brackets form part of the notation. The dimensions, or to give them their full title the dimensional formulae, of all other variables of interest in fluid mechanics can be expressed in terms of the four dimensions [M], [L], [T] and [Θ].

To introduce this notation, and the rules that operate, we consider a number of simple shapes. The area of a square, with sides of length l , is l^2 , and the dimensions of the square are $[L] \times [L] = [L \times L]$, which can be abbreviated to $[L^2]$. The area of a square, with sides of length $2l$, is $4l^2$. However, although the area of the second square is four times larger than that of the first square, the second square again has the dimensions $[L^2]$. A rectangle, with sides of length a and b , has an area ab , with dimensions of $[L^2]$. The area of a circle, with radius r , is πr^2 , with dimensions of $[L^2]$. While these figures are of various shapes and sizes, there is a common feature linking them all: they enclose a defined area. We can say that $[L^2]$ is the dimensional formula for area or, more simply, area has the dimensions $[L^2]$.

Let us consider a second example. If a body traverses a distance l in a time t , then the average velocity of the body over the distance is l/t . Since the dimensions of distance are [L], and those of time are [T], the dimensions of velocity are derived as $[L/T]$, which can also be written as $[LT^{-1}]$. By extending the argument a stage further, it follows that the dimensions of acceleration are $[LT^{-2}]$.

Since force can be expressed as the product of mass and acceleration the dimensions of force are given by $[M] \times [LT^{-2}] = [MLT^{-2}]$. By similar reasoning, the dimensions of any quantity can be quickly established.

1.2.8 Dimensional homogeneity

For a given choice of reference magnitudes, quantities of the same kind have magnitudes with the same dimensional formulae. (The converse, however, is not necessarily true: identical dimensional formulae are no guarantee that the corresponding quantities are of the same kind.) Since adding, subtracting or equating magnitudes makes sense only if the magnitudes refer to quantities of the same kind, it follows that all terms added, subtracted or equated must have identical dimensional formulae; that is; an equation must be *dimensionally homogeneous*.

In addition to the variables of major interest, equations in physical algebra may contain constants. These may be numerical values, like the $\frac{1}{2}$ in Kinetic energy $= \frac{1}{2} mu^2$, and they are therefore dimensionless. However, in general they are not dimensionless; their dimensional formulae are determined from those of the other magnitudes in the equation, so that dimensional homogeneity is achieved. For instance, in Newton's Law of Universal Gravitation, $F = Gm_1m_2/r^2$, the constant G must have the same dimensional formula as Fr^2/m_1m_2 , that is, $[MLT^{-2}][L^2]/[M][M] \equiv [L^3M^{-1}T^{-2}]$, otherwise the equation would not be dimensionally homogeneous. The fact that G is a universal constant is beside the point: dimensions are associated with it, and in analysing the equation they must be accounted for.

1.3 PROPERTIES OF FLUIDS

1.3.1 Density

The basic definition of the density of a substance is the ratio of the mass of a given amount of the substance to the volume it occupies. For liquids, this definition is generally satisfactory. However, since gases are compressible, further clarification is required.

Mean density

The *mean density* is the ratio of the mass of a given amount of a substance to the volume that this amount occupies. If the mean density in all parts of a substance is the same then the density is said to be *uniform*.

Density at a point

The density at a point is the limit to which the mean density tends as the volume considered is indefinitely reduced, that is $\lim_{v \rightarrow 0} (m/V)$. As a mathematical definition this is satisfactory; since, however, all matter actually consists of separate molecules, we should think of the volume reduced not absolutely to zero, but to an exceedingly small amount that is nevertheless large enough to contain a considerable number of molecules. The concept of a continuum is thus implicit in the definition of density at a point.

The *relative density* is the ratio of the density of a substance to some standard density. The standard density chosen for comparison with the density of a solid or a liquid is invariably that of water at 4 °C. For a gas, the standard density may be that of air or that of hydrogen, although for gases the term is little used. (The term *specific gravity* has also been used for the relative density of a solid or a liquid, but *relative density* is much to be preferred.) As relative density is the ratio of two magnitudes of the same kind it is merely a numeric without units.

Relative density

1.3.2 Pressure

A fluid always has pressure. As a result of innumerable molecular collisions, any part of the fluid must experience forces exerted on it by adjoining fluid or by adjoining solid boundaries. If, therefore, part of the fluid is arbitrarily divided from the rest by an imaginary plane, there will be forces that may be considered as acting at that plane.

Pressure

Pressure cannot be measured directly; all instruments said to measure it in fact indicate a difference of pressure. This difference is frequently that between the pressure of the fluid under consideration and the pressure of the surrounding atmosphere. The pressure of the atmosphere is therefore commonly used as the reference or datum pressure that is the starting point of the scale of measurement. The difference in pressure recorded by the measuring instrument is then termed the *gauge pressure*.

Gauge pressure

The *absolute pressure*, that is the pressure considered relative to that of a perfect vacuum, is then given by $p_{\text{abs}} = p_{\text{gauge}} + p_{\text{atm}}$. (See also Section 2.3.)

Absolute pressure

The pressure of the atmosphere is not constant. For many engineering purposes the variation of atmospheric pressure (and therefore the variation of absolute pressure for a given gauge pressure, or vice versa) is of no consequence. In other cases, however – especially for the flow of gases – it is necessary to consider absolute pressures rather than gauge pressures, and a knowledge of the pressure of the atmosphere is then required.

Pressure is determined from a calculation of the form (force divided by area), and so has the dimensions $[F]/[L^2] = [MLT^{-2}]/[L^2] = [ML^{-1}T^{-2}]$. Now although the force has direction, the pressure has not. The direction of the force also specifies the direction of the imaginary plane surface, since the latter is defined by the direction of a line perpendicular to, or *normal* to, the surface. Here, then, the force and the surface have the same direction and so in the equation

$$\overrightarrow{\text{Force}} = \text{Pressure} \times \overrightarrow{\text{Area of plane surface}}$$

pressure must be a scalar quantity. Pressure is a property of the fluid at the point in question. Similarly, temperature and density are properties of the fluid and it is just as illogical to speak of ‘downward pressure’, for example, as of ‘downward temperature’ or ‘downward density’. To say that pressure

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