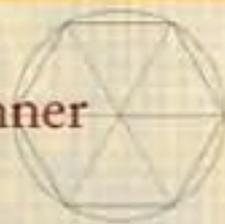


Numbers at Work

A Cultural Perspective



Rudolf Taschner



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TRANSLATED BY OTMAR BINDER
AND DAVID SINCLAIR-JONES



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The union of the mathematician with the poet, fervor with measure,
passion with correctness, this surely is the ideal.

—WILLIAM JAMES, *COLLECTED ESSAYS*

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Preface

Albert Einstein used to talk about the unique thrill he experienced when he looked over the shoulders of the *Old One*—his metaphor for the Creator, even though he had serious doubts about the existence of a personal God—and caught a glimpse of his playing cards.

But what did he mean by this? Ever since Pythagoras mathematicians have been certain that God’s “playing cards”—the building blocks of creation—are nothing other than numbers. This conviction was perhaps expressed most eloquently by Galileo:

The whole of Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to understand the language and to decipher the characters in which it is expressed. This language is mathematics, and its characters are triangles, circles, and other geometric figures. Without a knowledge of these it is humanly impossible to understand a single word of this book and we are condemned to traipse around aimlessly, lost in a dark labyrinth.

It is true that Galileo (and later Kant) saw mathematics as founded in geometry, which is itself directly grounded in what comes to us via our senses. This is still a hair’s breadth away from an actual peek into the cards of the Old One. Geometry does deprive sensory data of their color, corporeality, transience, vulnerability—in short, of all their opaque and baroque charms. Using straight lines, circles and their points of intersection, geometry attempts a representation of the bare bones of what we experience—what is left of our sensory data after abstraction has reduced it as far as it can. In geometry, the painter’s inspired brush strokes are reduced to the transparent, limpid constructions of an engineer wielding compass and ruler. Yet, geometry always retains an irreducible residue that affects our senses. Numbers do not.

In the case of arithmetic, the theory of numbers, there is no direct link connecting it to the experience of our senses. While we can see and touch the silver pieces we are counting and mentally connect them to the number thirty, we cannot see or touch the number thirty itself. We hear the

steeple clock strike, and we count up to eight—yet all that we hear is those strokes of the clock that we mentally subsume under eight; we do not hear the number eight itself. There is no way in which we can have direct sensuous experience of numbers, no way of experiencing them in their own right through any of our senses; neither our eyes nor our sense of touch nor our ears can give us access to them; they remain forever devoid of scent and taste.

We now know—and have known since David Hilbert’s insights into the foundations of geometry at the latest—that all of geometry can be incorporated into the realm of numbers. All the insights and conclusions of geometry can be deduced solely from the laws of arithmetic without recourse to sight or touch.

And it is not only geometry: this dictum may very well apply to all of intelligible reality. While this is not what this book purports to prove, an attempt is made in it to *point* towards the extent to which numbers are at work in a great variety of aspects of reality.

It is significant for the essential paradox of the human condition that it is numbers—cut off from sensuous experience by definition—that grant us our deepest insights into the nature of reality. Perhaps this is best illustrated by an anecdote that once more has Albert Einstein at its center.

As a young nobody Einstein loved to go to the zoo at Berne to watch the bear-feeding ritual. He noticed that the bears mostly trotted around with their snouts close to the ground, so that they only found those goodies they bumped into. However, sometimes one of them would rear itself up on its hind legs to gain a vantage point from which it could spot the choicest morsels. This reminded Einstein irresistibly of the majority of physicists, seeing no further than their noses as they crouch over their calculations. Truly significant discoveries are only made by those who survey the broad context. It is only by applying abstraction till it yields the numbers at the heart of things that we may hope to raise ourselves to such heights that we may sneak a peek at God’s playing cards.

RUDOLF TASCHNER

Translators' Note

Translating Professor Taschner's work proved a far more time-consuming labor than either of us had imagined when we first agreed to undertake this project. It was not the difficulty of rendering thoughts from German to English—we were familiar enough with that process and had allowed sufficient time for it. Still less was it any difficulty in liaison or interpretation. The problem was the work itself, and *problem* is perhaps not the word that best describes what delayed our arrival at the finishing post.

Reading *Numbers at Work* is like walking down a great corridor lined with books: there is much to see on the shelves, much to learn from each individual book, and great benefit to be gained from reaching the end. The trouble for the translator, and for all subsequent readers, is that the corridor is studded with doors, each of them leading into a corridor of its own, with shelves and books and doors of its own. It is very difficult to press ahead without investigating at least some of these distractions. We strongly advise you not to try—to give in to temptation and to follow each tantalizing lead: there is, as Galileo pointed out, a whole universe to explore.

OTMAR BINDER

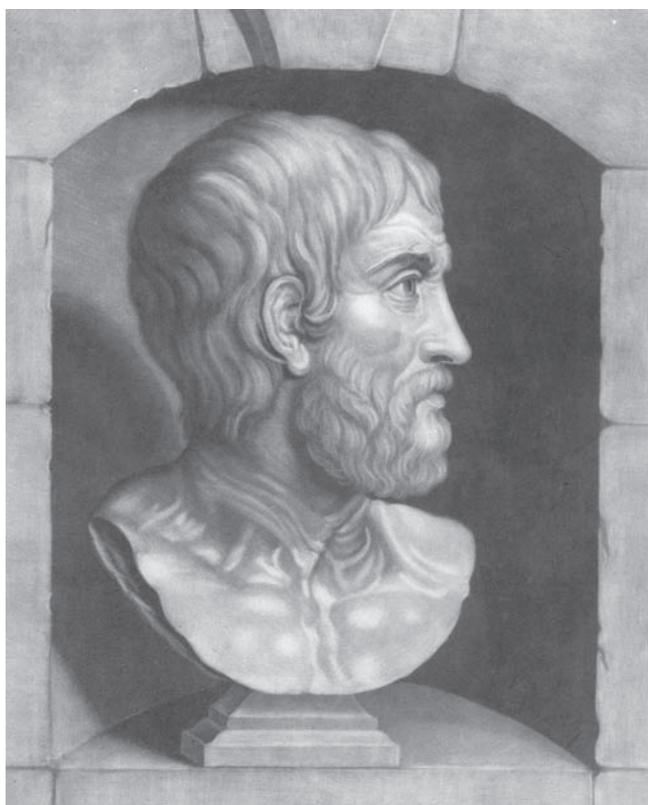
DAVID SINCLAIR-JONES

Pythagoras

Numbers and Symbol

Numbers are the key to all knowledge, according to Pythagoras of Samos. He believed that they underlie the whole cosmos and all that is in it, our very selves and all that goes to make us what we are. Nothing exists that cannot be expressed as numbers.

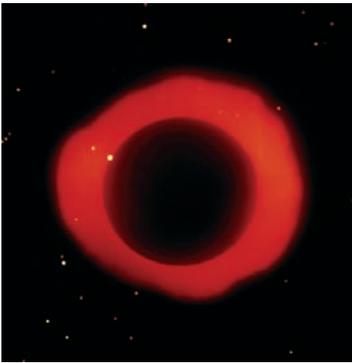
Nobody knows how Pythagoras arrived at this idea. He wrote nothing—or at least nothing that has come down to us. Nonetheless, there is enough in the historical record to allow us some room for conjecture.



1 Pythagoras of Samos.



2 Thales of Miletus.



3 His knowledge of Babylonian astronomy enabled Thales to predict the solar eclipse of 585 BC.

Thales of Miletus, Pythagoras' elder by less than a generation, lived around 600 BC. Pythagoras traveled widely, always seeking out experiences and people who could help him in his studies. It is very possible that he met Thales, and it is almost certain that he was influenced by the man who is still seen as the prototypical philosopher.

Thales' teaching career can be dated with reasonable precision. He is credited with predicting the solar eclipse of May 28, 585 BC. This feat was stunning enough as a purely scientific achievement. What is said to have followed is surely one of the most important developments in the intellectual history of mankind.

At the precise moment of this eclipse, a battle was raging between armies led by Alyattes of Lydia and Cyaxares of the Medes. The Medes, shocked by the sudden switch from day to night, dropped their arms and fled, leaving the victory to Alyattes, who held his ground. Alyattes had been forewarned of the celestial phenomenon by Thales.

We may doubt the historical authenticity of this account,¹ but the message it conveys is unique as a turning point

in the intellectual history of mankind. Rational thinking applied to experience enabled Thales to predict the eclipse; foreknowledge of the eclipse helped Alyattes to victory. The losers, hobbled by ignorance, put the eclipse down to the wrath of the gods and fled. The victors, knowing it to be a natural phenomenon brought about by an understandable sequence of events, held their ground and won. It was not the whim of the gods, creatures of myth, that helped Alyattes to victory; rather, it was the conclusions of Thales, drawn solely from what the Greeks called *lógos*.²

Thales, then, is a likely source for this key insight in Pythagoras' thought: ours is not the role of inert playthings at the mercy of irrational, unfathomable divine forces. The universe is not chaos running

riot; rather, we are part of an orderly cosmos that awaits our rational interpretation.

Perhaps³ it was at this point that Pythagoras raised an obvious, yet at the same time, all-decisive question: What is the basis of understanding? What are, as it were, its basic building blocks, its atoms? Where does it all begin? What is so simple and so self-evident that no further explanation is needed? What are the axioms it would be pointless to question, because everything about them has been established beyond doubt?

Pythagoras came to believe that he had an answer to that question. Nothing, as he saw it, was more elemental than counting. For someone who had grasped the principle of counting, beginning with *one* and working up to ever-new numbers by continually adding *one*, it was simply inconceivable to proceed in any other way. Counting is an activity in which all humanity, its manifold differences in all other areas notwithstanding, behaves as one on a global scale. In fact, it is generally agreed today that if radio contact with intelligent extraterrestrials were possible,⁴ this contact would have to be couched in the terms of counting. Counting, in effect, is seen as the only technique common to all intelligent beings.

Here, then, is the *axiom*: only when we understand what numbers are at the core of a certain state of affairs can we be certain that we actually understand it. To understand something completely means to understand it as self-evidently as we understand counting.

Thales leads us to believe that we can understand the world. It follows, for Pythagoras, that the world must exist as numbers, because it is only when a state of affairs is reduced to numbers that it can be understood.

One may object that there are civilizations that are not acquainted with the idea of numbers. Some peoples—for instance, the *Abipones* and the *Yonoama* in Latin America or the *Rumilara* in Southern Australia—are effectively innumerate, capable of only grasping ideas of entities in the singular, in pairs and, at the very most, in triads. Confronted with more than three items, Brazilian *Bakairi* or *Bororo* Indians simply see a *multitude* and grab their hair to express that diagnosis. For them, the question “How many precisely?” is simply not one that comes to mind.

We can furthermore object that we simply do not know how our understanding of numbers has come about. After all, is not the discovery of numbers hidden in the dawn of history?

To begin with, numbers were tied inseparably to the objects to be counted. In early Mesopotamian times, two different *fours* were needed for *four bushels of wheat* and *four beef carcasses*. Vestiges of this proto-understanding of numbers have been preserved until today. Having tried on *two* shoes, we buy the *pair*—we make a distinction between *pair*, which



4 Early Sumerian numerical and script characters.

denotes two-ness at a stage that precedes counting, and *two*, a full-fledged number. Number as an idea, number emancipated from the type of object being counted, seems to have emerged in Mesopotamia as early as the ninth century BC. If a trader took to the road with goods—say, five heads of cattle and seven sheep—he would have carried a box containing a representation of these goods, perhaps five clay balls and seven tablets. To be on the safe side, the contents of the box may have been displayed in pictures on the outside—this at least is how we interpret certain archaeological finds dating from the period. The next step was the realization that the pictures themselves provided sufficient information: a big step on the road toward script characters. Early Sumerian numerical symbols and script characters indeed show a close resemblance to these pictures. The ability to count seems to be inextricably intertwined with the skills of verbalizing, writing and reading.

Leaving these objections aside, substantial though they may be, it is clear that Pythagoras became convinced he could *prove* that numbers are the basic building blocks of everything that exists. He had discovered that dividing a single string (on an instrument called the *monochord*) into ratios of small whole numbers, such as 1:2, 2:3, 3:4, etc., generated musical intervals. These intervals, when used melodically, were capable of moving listeners to tears and speaking directly to their souls. This insight is laid down in the wonderful words of one of his pupils, Philolaus: “The soul is the numerical harmony of the body, and the soul’s relationship to the body is the same as the one existing between notes and the musical instrument that produces them.”

5 The wall of the “geometry room”, Nicolaus Copernicus’ study in Cracow, is full of sketches of the planets with their relative orbits.



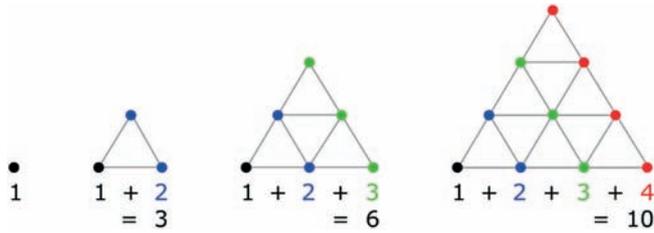
Even celestial bodies were seen as restricted in their movements to spheres whose relationships, according to Pythagoras, were based on numerical ratios. Legend has it that these movements so enthralled Thales that his enthusiastic study of them once led to his falling into a well. These ratios of small whole numbers produced chords of a beauty that was held to be quite literally supernatural in that they could only be perceived by the gods: what has been called the *music of the spheres*.

The idea of such planetary spheres survived into the modern age. As late as 1596, a 25-year-old surveyor by the name of Johannes Kepler published his *Mysterium cosmographicum* (*The Secret of the Universe [Explained]*) in Graz, then the capital of the Habsburgs' southern possessions. In this book, he accounts for the distances between the sun and the planets by postulating a mutual interpenetration of the five regular Platonic bodies. These bodies determine the radii of the calottes or spherical surfaces that support the planets. By 1609, in his *Astronomica nova* (*The New Astronomy*), Kepler himself had discarded this concept, one of the legacies of antiquity, and cleared the way for the modern explanation of celestial mechanics.

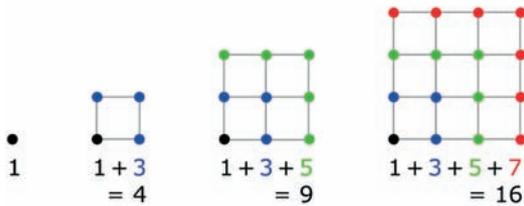
But let us return once more to antiquity. It is not surprising that Pythagoras, in the euphoria of a great discovery, drew a great number of inferences from it, the majority of which were misconceived (the idea of the music of the spheres being one example) and led only to further errors. Certainly the inspired number symbolism of the Pythagoreans, like the numerologies of other cultures, stayed at surface level and did not penetrate to a deeper understanding. Before we try to explain why

this was so, let us spend a little time in the company of those who have tried to seek knowledge through the symbolic power of numbers.

Following age-old tradition, Pythagoras attempted to identify geometrical patterns related to numbers. He meditated on triangular numbers



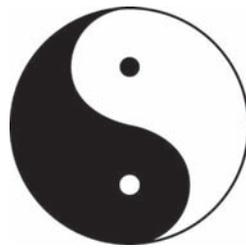
and so on, on square numbers



and so on, and on similar numerical patterns. He differentiated between *even* numbers, which can be arranged in half as many pairs, and *odd* numbers, the pairing of which always leaves a remainder of one. Interestingly enough, for some advocates of numerical symbolism, odd numbers are considered to be “good”; even numbers, “evil”. This may be because if odd numbers are added one to another in sequence beginning with *one*, the answers give the square numbers in sequence: $1 + 3 = 4$, $1 + 3 + 5 = 9$, $1 + 3 + 5 + 7 = 16$, and so on.⁵ No such relationship exists for even numbers.

Geometrical patterns such as those mentioned earlier were seminal for numerical symbolism. It is interesting to note that even in antiquity, *one* had a special status among numbers: it was regarded as the unit and did not normally figure in counting. This is why *one* was used as the symbol of the indivisible and, ultimately, of the divine.

For the ancient Greeks, *two* was the first genuine number. It symbolized opposition, antithesis, disunion: the polarity of man and woman, of left and right, of good and evil, active and passive, sun and moon, day and night.



6 Yin and Yang.

In the Christian tradition, this motif is taken up by the polarity of the Old and the New Testament as well as that of man and God—foreshadowing Jesus, who, according to Christian theology, unites these two natures—human and divine—in himself.

Judaic tradition shows its own familiarity with the symbolic significance of the number two, which can most clearly be seen in the two tablets that Moses received for his people from God.

Three, the typical triangular number (and the first after *one*), symbolizes a kind of completeness in its geometrical pattern: the area of a triangle, which is defined by reference to its three vertices. This completeness is closely related to the idea of perfection. The trinity of mother, father and child has been seen from ancient times as a blueprint for human social life. In antiquity, this trinity often took the form of divine triads: Anu, Enlil, Ea in ancient Babylon; Brahma, Vishnu, Shiva in India; and, of course, the Christian trinity of Father, Son and Holy Spirit. In the Old Testament, the number *three* is of symbolic significance in several passages. In Isaiah, for instance, we find the following, in the words of the *Authorized Version*:

In the year that king Uzziah died, I saw also the Lord sitting upon a throne, high and lifted up, and his train filled the temple. Above it stood the seraphims: each one had six wings; with twain he covered his face, and with twain he covered his feet, and with twain he did fly. And one cried unto another, and said, Holy, holy, holy is the Lord of hosts: the whole earth is full of his glory.

In Genesis, Abraham is visited by God—the one and only God—in the shape of three men. This happens in a mysterious manner, which the text expresses by shifting from three to one:

And the Lord appeared unto him in the plains of Mamre; and he sat in the tent door in the heat of the day;

And he lifted up his eyes and looked, and, lo, three men stood by him; and when he saw them, he ran to meet them from the tent door, and bowed himself to the ground,

And said, My Lord, if now I have found favour in thy sight, pass not away, I pray thee, from thy servant....



7 Trinity of mother, father and child.



8 Abraham is visited by God in the guise of three men.



9 Compass rose.

And Genesis describes the creation of man thus:

So God created man in his own image,
in the image of God created he him,
male and female created he them.

The perfection of creation is emphasized by the threefold repetition of “created” in three lines.

Four is the typical square number (again, the first such number after *one*). It symbolizes cosmological constructs: the four points of the compass, the four wind directions, the four seasons and Empedocles’ four elements of fire, water, earth and air. The Bible gives us the four rivers Pishon, Gihon, Tigris and Euphrates that encircle the Garden of Eden and Daniel seeing in his dream four beasts (symbolizing the rulers of the global empire) emerging from a sea lashed by four winds.

The symbolic significance of the numbers *three* and *four* is further underlined in the Bible by the fact that the Israelites had three patriarchs—Abraham, Isaac and Jacob—and four archetypal mothers—Sarah, Rebecca, Leah and Rachel.

Seven (the sum of *three* and *four*) and *twelve* (their product) are themselves of immense numerological importance.

Seven stands as the number of planets (in the ancient sense of heavenly bodies that change their position with reference to the fixed stars) visible to the naked eye: Sun, Moon, Mercury, Venus, Mars, Jupiter and Saturn. In this way, it evokes the perfection of the universe. The Old Testament is replete with references to the number *seven*, and Christianity has continued this tradition. Consider the seven loaves of bread with which Jesus feeds the four thousand, and the seven baskets full of scraps left over (Matt. 15:34–37).⁶ Consider the seven petitions in the “Our Father”, the seven sacraments, the seven virtues, the seven deadly sins, the seven gifts

of the Holy Spirit and the seven works of charity.

Twelve is the number of the zodiac in the nocturnal sky of the Northern Hemisphere; the ancient Egyptians subdivided the day into twelve hours and the year into twelve months; the Gnostics proclaim twelve eons. There are twelve tribes of the Children of Israel; the Church as the New Israel acknowledges twelve apostles; in the Old Testament Book of Judges, the number of great judges is also twelve.

Doubling numbers strengthens their potency as symbols. In the Isaiah verses quoted earlier, the seraphs have six wings arranged in three pairs. The number *six* is the next triangular number after *three*. It is also the result of doubling *three*. Hence, its geometrical pattern is that of the *Magen David*, the Shield (or Star) of David, which consists of two equilateral triangles. Analogous to this is *eight*, whose geometrical pattern is the octagon, two superimposed squares, one of which has been rotated by 45°, each of them symbolizing the typical square number *four*. The octagon is a recurring feature in architectural design and in the arts. In Buddhism, a circle subdivided into eight segments, which derives from the octagon, is an emblem of the eight different paths on which to escape from the vale of tears of imperfect existence. Murray Gell-Mann, winner of the 1969 Nobel Prize for Physics, discovered a new and completely unexpected significance for *eight*: it represents the family of elementary particles to which protons and



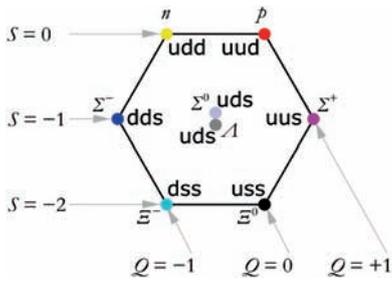
10 In the ancient and medieval cosmos, the seven *planets*—the Moon, Mercury, Venus, the Sun, Mars, Jupiter and Saturn—and the twelve constellations go around the Earth.



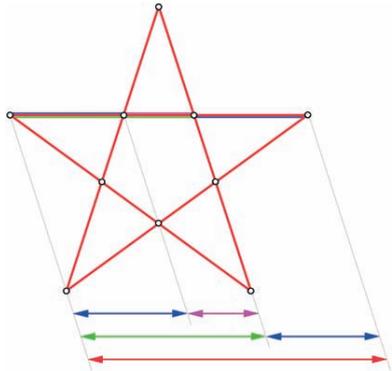
11 The Shield of David.



12 The octagon in the dome of Aachen.



13 The octet of heavy particles according to Gell-Mann.



14 The diagonals of a regular pentagon form a pentagram. They intersect with one another so that the ratio between the entire length and the longer section is the same as that between the longer section and the medium one and between the medium section and the short one. This threefold ratio is the *Golden Mean*.

15 The cathedral of Notre Dame has many examples of the *Golden Mean*. Examples are the ratio between the total width and the width of one of the towers, the ratio between the height of the entrance story and that of the middle story with the rosette and the ratio between the height of the middle story to the top story.

neutrons, the constituent components of the nucleus, belong.

Five generates the pentagram; for the Pythagoreans, this was the most mysterious geometrical pattern of them all. If you draw the five diagonals in a regular pentagon, a new regular pentagon appears in the middle, where again the diagonals can be drawn and so *ad infinitum*. The line sections produced by the intersecting diagonals form an aesthetically highly satisfying ratio, which later came to be called the *Golden Mean*. The Golden Mean appears in countless proportions in the fine arts and in architecture. In addition to this, the task of defining the Golden Mean by means of a numerical proportion proved an insoluble problem for Pythagoras' pupils. It was even insoluble for reasons of principle, as they noted to their astonishment, but this is discussed in greater detail later.



Forty, in itself a number of no particular numerological interest, still has a very powerful symbolic significance in the broad context of purification, which survives to this day in the etymology of the word *quarantine*. We should also cite the belief that the fortieth day marks the end of an acute illness and the division of pregnancy into seven periods of forty days. *Forty* features prominently in Jewish and Islamic mysticism, in the Hebrew Bible and in the New Testament.⁷ Some examples are the Israelites' forty years wandering through the wilderness (Num. 32:13), the duration of the Flood (Gen. 7:17), Moses' two sojourns on Mt. Sinai (Exod. 24:18) and Jesus fasting in the desert (Matt. 4:2).

Numbers that refused to be harnessed in geometrical patterns, or "refractory" numbers, were of particular interest to the Pythagoreans. The best known among these are the *prime numbers*. It is impossible to represent such numbers in rectangular patterns: 2, 3, 5, 7, 11, 13 or 17 dots can only be arranged in one line for rectangular presentation, whereas 9, 12 and 15 can be arranged in two or more equal lines.



The horizontal and the vertical extension of a rectangular number (i.e., the number of dots in the two directions) are obviously divisors of this number. The preceding figure shows that 9 has the divisor 3, 12 has the divisors 4 and 3 (and 6 and 2 as well), and 15 has the divisors 5 and 3. Prime numbers, on the other hand, have only one and themselves as possible divisors.⁸ Even numbers always have two as a divisor (and are therefore never—with the exception of two itself—prime numbers).

For a number like $31 (2 \cdot 3 \cdot 5 + 1)$, 2, 3 or 5 are not possible divisors. If you try to express 31 as a rectangular pattern with one of these prime numbers as its horizontal extension, there will be a remainder of 1. Similarly, a number like $211 (2 \cdot 3 \cdot 5 \cdot 7 + 1)$ cannot have either 2, 3, 5 or 7 as divisors. Divisions by one of these prime numbers must again always leave a remainder of 1. Calculations like these led Euclid to conclude that no list of prime numbers, no enumeration of a *finite* number of prime numbers, was ever going to be exhaustive.⁹

Today we tend to say rather grandiosely, "There is an infinite number of prime numbers," but no equally grandiose proof is available. There is no magician to whisk a silk cloth off the table and reveal infinite cohorts of prime numbers happily disporting themselves!

In addition to geometrical patterns being generated by numbers, there is the attractive reverse procedure: arranging numbers in such a way that they can be fitted into geometrical patterns. Going back at

least as far as Pythagoras' days, we find a number of different civilizations experimenting with "magic squares". A magic square consists of $3 \cdot 3 = 9$, $4 \cdot 4 = 16$ boxes or a square number of boxes arranged within a square. Each square has to be filled with the numbers $1, 2, 3, \dots, n$ (where n denotes the number of boxes in the square) in such a way that every row, every column and the two diagonals add up to the same sum.

In China, a magic square made up of three rows and three columns called *Lo-Shu* is still sold today as a lucky charm. The sum in every row and every column and in the two diagonals must add up to 15, so we call 15 the *magic number* of any 3×3 square. This follows from the fact that the sum of the numbers 1 to 9 is 45. When this is divided into three equal addends, 45 yields 15, which therefore emerges as the *Lo-Shu* magic number. There are only a few ways to break down 15 as the sum of three numbers between 1 and 9; the following table gives a systematic survey:

4	9	2
3	5	7
8	1	6

16 *Lo-Shu*.

$$\begin{array}{cccc} 9 + 5 + 1 = 15 & 8 + 6 + 1 = 15 & 8 + 4 + 3 = 15 & 7 + 5 + 3 = 15 \\ 9 + 4 + 2 = 15 & 8 + 5 + 2 = 15 & 7 + 6 + 2 = 15 & 6 + 5 + 4 = 15 \end{array}$$

All of these are realized in the *Lo-Shu*, this is why there are no 3×3 magic squares—with the exception of simple mirror inversions—found outside the *Lo-Shu*.

The magic number of a 2×2 square is $5 = (1 + 2 + 3 + 4)/2$. However, 5 can only be broken down into two addends ($4 + 1$ and $2 + 3$), which cannot provide us with a 2×2 magic square.



17 The magic square is above the angel's wing in Dürer's *Melencolia*.

The magic number of a 4×4 square can be found by the formula

$$1 + 2 + 3 + \dots + 14 + 15 + 16 = 136$$

and is therefore $34 (= 136/4)$. There are 86 possible ways to arrive at this sum using four addends between 1 and 16. It is not surprising, therefore, that there are many variations¹⁰ on the 4×4 magic square. The most famous magic square is found in one of Albrecht Dürer's engravings, his 1514 work "Melencolia", which was occasioned by the death of the artist's mother in the same year. The artist has depicted a magic square in the upper right corner of the picture. Dürer con-

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