CliffsQuickReview® Precalculus

By W. Michael Kelley

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INTRODUCTION

CliffsQuickReview Precalculus is a comprehensive volume of the topics usually included within a course intended to serve as a calculus prerequisite. Although the collection of skills deemed worthy of inclusion in such a course may vary slightly from instructor to instructor, this text contains all of the most commonly discussed elements, including:

- Arithmetic and algebraic skills
- Functions and their graphs
- Polynomials, including binomial expansion
- Right and oblique angle trigonometry
- Equations and graphs of conic sections
- Matrices and their application to systems of equations

It is assumed that you have some knowledge of algebra and its concepts, although nearly all of the foundational algebraic skills you'll need are reviewed in the early chapters of this book. If you feel you need to further review these concepts, refer to *CliffsQuickReview Algebra I* and *Algebra II*.

Why You Need This Book

Can you answer yes to any of these questions?

- Do you need to review the fundamentals of precalculus?
- Do you wish you had someone else to explain the concepts of precalculus to you other than your teacher?
- Do you need to prepare for a precalculus test?
- Do you need a concise, comprehensive reference for precalculus?

If so, then *CliffsQuickReview Precalculus* is for you!

How to Use This Book

This book puts you in the driver's seat; use it any way you like. Perhaps you want to read about the topics you'll learn in class before your teacher discusses them, so that you have a leg up on your classmates and stand a better chance to understand since you'll already have an idea about what's going on. Maybe you want to read the book cover to cover, or just consult it when you're having trouble understanding what's going on in class. Either way, here are a few ways you can search for more information about a particular topic:

- Use the Pocket Guide (the tear-out card in the front of the book) to find essential information, such as key formulas and concepts.
- Look for areas of interest in the Table of Contents, or use the index to find specific topics.
- Flip through the book, looking for subject areas at the top of each page.
- Get a glimpse of what you'll gain from a chapter by reading through the "Chapter Check-In" at the beginning of each chapter.
- Use the Chapter Checkout at the end each chapter to gauge your grasp of the important information you need to know.
- At the end of the book, you can test your knowledge more completely in the CQR Review at the end of the book and look for additional sources of information in the CQR Resource Center.
- Use the glossary to find key words quickly. Terms are written in **boldface** when first introduced in the book, so their definitions are always close by. In addition, all of the important boldface terms are defined in the book's glossary.

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Chapter 1
PRECALCULUS PREREQUISITES

Chapter Check-In

- ❑ Defining common number groups
- ❑ Writing inequalities as intervals
- ❑ Understanding algebraic properties
- ❑ Working with exponents, radicals, polynomials, and rational expressions
- ❑ Finding solutions to equations and inequalities
- ❑ Constructing linear equations

A strong algebraic background is essential to success in precalculus. Before you can begin exploring its more advanced topics, you must first have a firm grip on the fundamentals. In this chapter, you'll review and practice foundational concepts and skills.

Classifying Numbers

Many times throughout your precalculus course, you'll be manipulating specific kinds of numbers, so it's important to understand how mathematicians classify numbers and what kinds of major classifications exist. Be aware that numbers can fall into more than one group. Just as an American citizen can also be classified as a North American citizen or a citizen of Earth, numbers may also belong to numerous categories simultaneously.

The following groups, or sets, of numbers are generally agreed on by mathematicians as the most common classifications of numbers. They are listed here in order of size, from smallest to largest:

- **Natural numbers.** The set of numbers you've used since you were very young when counting (as such, the *natural numbers* can also be called the *counting numbers*): {1, 2, 3, 4, 5, 6, ...}.

- **Whole numbers.** The *whole numbers* include all of the *natural numbers* and, also, the number 0: {0, 1, 2, 3, 4, 5, ...}.
- **Integers.** All of the whole numbers and their opposites make up the set of *integers*. In other words, any number without an extra decimal or fraction attached to it is considered an integer: {..., –3, –2, –1, 0, 1, 2, 3, ...}. Because *integers* contain no obvious fractions or decimals, some students are tempted to refer to them as *whole numbers*, but that is not completely accurate, because the set of *whole numbers* does not include negative numbers.
- **Rational numbers.** A number is classified as *rational* if one of the following conditions hold true.

  The number can be expressed as a fraction. (In other words, the number can be rewritten as $\frac{a}{b}$, where a and b are integers, and $b \neq 0$.)

  The number is a terminating decimal, in other words a decimal that ends (such as 6.25) rather than continues on infinitely.

  The number is a decimal that repeats in an infinite pattern (such as 5.297297297297...).

  Basically, any number that can be written as a fraction is *rational*.

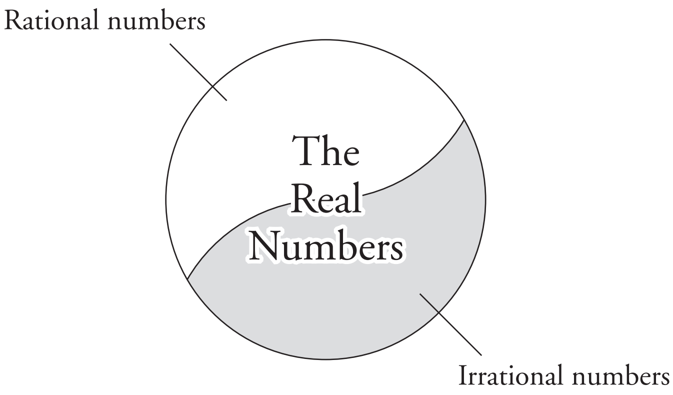
Example 1: Show that any integer must also be a rational number.

Any integer a can also be rewritten as $\frac{a}{1}$, since dividing by 1 will not alter the value of the integer. Because a can be expressed as a fraction whose numerator and denominator are both integers, a must be rational by definition.

- **Irrational numbers.** A number that cannot be written as a fraction is considered *irrational*. The most obvious indicator of an irrational number is a decimal that doesn't infinitely repeat itself yet never terminates. For example, π is an irrational number whose decimal equivalent 3.14159265359... never ends and never follows any obvious repeating pattern. Many radicals, like $\sqrt{3}$, are irrational numbers.
- **Real numbers.** The *real numbers* are made up of the *rational numbers* and the *irrational numbers* grouped together, as shown in Figure 1-1.
- **Complex numbers.** *Complex numbers* differ from the *real numbers* in appearance quite starkly. Complex numbers usually have two distinct parts and look like $a + bi$, where a is the *real part*, bi is the *imaginary part*, where i is equal to the imaginary value $\sqrt{-1}$. However,

numbers need not contain both parts to be considered complex. In fact, any real number is automatically complex. For example, since the real number 3 can be written as $3 + 0i$, 3 is a complex number. It just contains no imaginary part.

Figure 1-1 The rational and the irrational numbers together comprise the entire set of real numbers. Note that this drawing is not to scale. Far more irrational than rational numbers exist.



Interval Notation

Traditional inequality statements can be rewritten using **interval notation**, a shorthand method that expresses the same meaning but usually in a more compact and intuitive manner. This is largely due to the fact that interval notation clearly defines the boundaries of the inequality with which you're working.

Bounded intervals

If you're given an inequality that is bounded on both sides by a real number, that statement can be rewritten as a **bounded interval**. To create a bounded interval, write the two numerical endpoints of the interval in order, always from lowest to highest. (The interval will almost look like a coordinate pair.) Then, indicate whether that point should be included on the interval. If it should, use a bracket with that endpoint; if it should not, use a parenthesis.

Example 2: Rewrite the following inequality statements using interval notation.

(a) $-5 \le x \le 3$

Because the inequality signs stipulate less than *or equal to*, you must include the endpoints in the interval. Had equality not been a possibility, you would not include those endpoints. Use brackets to indicate inclusion: [–5,3].

(b) $1 > x > 0$

Even though this interval is written so that the upper boundary is on the left, interval notation still requires you to write them in order from lesser to greater. Use parentheses to indicate that the endpoints are not included in the interval: (0,1).

(c) $-2 \le x < 4$

The lower endpoint is included while the upper is not: [–2,4).

If both endpoints of the interval are included (as in part [a] of Example 2), the interval is said to be **closed**. On the other hand, if neither endpoint is included (as in part [b] of Example 3), it is an **open interval**.

Unbounded intervals

Sometimes, only one endpoint of an interval is explicitly defined and the other is implied. For instance, consider the inequality $x > 3$. Clearly, the lower boundary of the interval is 3, but what is the upper boundary? Because there is no finite value given for the upper endpoint, you use infinity. If one or more of the endpoints of an interval are understood to be infinite, the interval is said to be **unbounded**.

You will use two different infinite boundaries:

- ∞, if the boundary is infinitely positive (it is used as the upper bound of the interval)
- $-\infty$, if the boundary is infinitely negative (it is used as the lower bound of the interval)

Infinity is technically not a real number, which means you can never use a bracket to indicate its inclusion in the interval. Instead, always use a parenthesis.

Example 3: Rewrite the following inequality statements using interval notation:

(a) $x > -1$

The lower bound of the interval is –1, and the upper bound is infinitely large, since any positive number will make this inequality statement true. The lower boundary should not be included, because the relationship is "greater than," not "greater than or equal to": $(-1,\infty)$.

(b) $x \leq 3$

In this interval, 3 is the upper boundary. If the lower boundary of an interval is infinite, you must indicate this by using negative infinity: $(-\infty,3]$.

(c) All real numbers

Any real number, from the infinitely negative to the infinitely positive, should be included in this interval: $(-\infty,\infty)$.

Algebraic Properties

Properties, also called *laws* or *axioms*, are foundational mathematical principles that are assumed true. Although there is no way to irrefutably prove properties, they make enough inherent common sense to be universally agreed on by mathematicians. It's a good thing they are, because these laws form the backbone of algebra.

The associative property

Given a string of numbers added together, you may group the numbers in any order you wish and it will not affect the answer you get. This is the basic premise of the **associative property** for addition. In other words, no matter what numbers are *associated* together, you will get the same result in the end.

$$(1 + 3) + 5 = 1 + (3 + 5)$$
$$4 + 5 = 1 + 8$$
$$9 = 9$$

The associative property also holds true for multiplication, but it fails for both subtraction and division. Here are the official mathematical definitions for its two incarnations:

- **The associative property for addition:**

  $(a + b) + c = a + (b + c)$

- **The associative property for multiplication:**

  $(a \cdot b) \cdot c = a \cdot (b \cdot c)$

Note that the symbol · is used here to indicate multiplication rather than the other traditional symbol for multiplication, ×. This is because it's easy to confuse the operation × with the variable *x* when you're working a problem.

The commutative property

This property (like its sister, the associative property) works only for addition and multiplication. In essence, it says that given a string of numbers being added or a string of numbers being multiplied, the order in which you complete that operation doesn't matter.

$$3 \cdot 9 = 9 \cdot 3$$
$$27 = 27$$

- **The commutative property for addition:**

  $a + b = b + a$

- **The commutative property for multiplication:**

  $a \cdot b = b \cdot a$

The distributive property

According to the **distributive property**, if terms are being added or subtracted within parentheses and a number appears "outside" that group of terms, you can multiply that outer number through to every number within those parentheses.

$$a(b + c) = ab + ac$$

Example 4: Rewrite using the distributive property:

$$3(x - 7)$$

Multiply every term in the parentheses by 3:

$$3 \cdot x - 3 \cdot 7$$
$$3x - 21$$

Identity elements

Fixed numbers called **identity elements** exist for both the operations of addition and multiplication. These elements do not alter a number's value (or *identity*) when the operation is applied to them. The *identity element for addition* (also called the *additive identity*) is 0, because if you add 0 to any number, you get back what you started with:

$$2 + 0 = 2$$

Similarly, the *identity element for multiplication* (also called the *multiplicative identity*) is 1, since multiplying any number by 1 will not change that number's value:

$$3 \cdot 1 = 3$$

These identity elements are important because they are a major component in the *inverse properties.*

Inverse properties

Once again, the operations of addition and multiplication have a property specific to them. In both cases, an **inverse property** assures you that no matter the input, there is a way to "cancel it out."

- **Additive inverse property:** For any real number a, there exists a real number $-a$ (the *opposite* of a) so that $a + (-a) = 0$:

$$4 + (-4) = 0$$

- **Multiplicative inverse property:** For any non-zero real number a, there exists a real number $\frac{1}{a}$ so that $a \cdot \frac{1}{a} = 1$:

$$7 \cdot \frac{1}{7} = 1$$

Note that when you "undo" addition and multiplication using these inverse properties, the result will be the *identity element* for the corresponding operation.

Exponential Expressions

Repeated multiplication can be rewritten using **exponents**, small numbers written above and to the right of the **base** number, both to clarify and simplify your notation. Rather than write "$x \cdot x \cdot x$," you can write "x^3," which is read "x to the third **power**." The *power* of an exponent is the number of times the object is multiplied by itself. Therefore, in the expression x^3, x is considered the *base* and 3 is the *power*.

There are six important rules you should know when undertaking any arithmetic involving exponents:

- **Rule 1:** $x^a \cdot x^b = x^{a+b}$

  If two exponential expressions with identical bases are multiplied, the result is that base raised to an exponent equal to the sum of the two powers:

  $$x^4 \cdot x^7 = x^{4+7} = x^{11}$$

- **Rule 2:** $\frac{x^a}{x^b} = x^{a-b}$

  If two exponential expressions with identical bases are divided, the result is that base raised to an exponent equal to the power in the numerator minus the power in the denominator:

  $$\frac{x^8}{x^5} = x^{8-5} = x^3$$

- **Rule 3:** $\left(x^a\right)^b = x^{a \cdot b}$

  If an exponential expression is itself raised to a power, the result is the base raised to the product of the two powers:

  $$\left(x^2\right)^6 = x^{2 \cdot 6} = x^{12}$$

- **Rule 4:** $\left(x^a y^b\right)^c = x^{ac} y^{bc}$

  If numerous exponential factors are raised to a power, multiply the outer power times each of the inner powers.

  $$\left(x^2 y^5\right)^3 = x^{2 \cdot 3} y^{5 \cdot 3} = x^6 y^{15}$$

- **Rule 5:** $x^{-a} = \frac{1}{x^a}$ and $\frac{1}{x^{-b}} = x^b$

  A negative exponent indicates that the expression is in the wrong part of the fraction. To make the exponent positive again (no algebraic expression is completely simplified until it contains no negative exponents), move the exponential expression to the other side of the fraction bar. For instance, if it is in the numerator, move it to the denominator, and leave the base alone.

  $$\frac{x^{-2}}{y^{-3}} = \frac{y^3}{x^2}$$

- **Rule 6:** $x^0 = 1$ (if $x \neq 0$)

  Any real number raised to the 0 power is equal to 1 (with the exception of 0^0, which does not have a real number value).

Example 5: Simplify using the exponential rules:

(a) $\dfrac{x^3y^5z^2}{xy^7z^2}$

Rewrite the fraction using Rule 2. Since the x in the denominator has no visible exponent, it is understood to be 1.

$$x^{3-1}y^{5-7}z^{2-2}$$

$$x^2y^{-2}z^0$$

A completely simplified solution does not contain negative exponents. Apply Rule 5 to achieve that goal. In addition, rewrite z^0 as 1.

$$\frac{x^2}{y^2}$$

(b) $(x^2y^3)(x^7yz^3)$

You can rearrange the terms thanks to the commutative propery and then add exponential powers of like bases, thanks to Rule 1. Again, since the y in the second group of parentheses has no exponent visible, it is understood to be 1.

$$x^{2+7}y^{3+1}z^3$$

$$x^9y^4z^3$$

(c) $\left(\dfrac{x^2y^{-2}}{z^{-3}}\right)^{-2}$

Begin by applying Rule 4:

$$\frac{x^{-4}y^4}{z^6}$$

Use Rule 5 to eliminate negative exponents:

$$\frac{y^4}{x^4z^6}$$

Radical Expressions

Although most of the time the exponents you'll see will be integers, you may run across some fractional powers as well. These types of powers translate into **radicals** (also called *roots*):

$$x^{a/b} = \sqrt[b]{x^a} \text{ or } \left(\sqrt[b]{x}\right)^a$$

You can use either notation to rewrite the fractional power as a radical. In some cases, one form will be more useful than the other when you are simplifying.

A typical radical, $\sqrt[n]{x^a}$ contains two parts: the **index** (the small number in front of the radical) and the **radicand**, the quantity within the radical symbol. It is read "The *n*th root of *x* to the *a*th power." Note that if no *index* is given for the radical, the *index* is understood to be 2.

Some students find radicals easier to understand if they think of the notation as a question. For example, the radical $\sqrt[3]{8}$ asks the question "What number multiplied by itself 3 times is equal to 8?" The answer is 2, so $\sqrt[3]{8} = 2$.

Properties of radicals

Because radicals are really exponents in disguise (even if they are fractional exponents), radicals possess the same properties as exponents. In addition, radicals have these properties:

- $\sqrt[n]{x^a y^b} = \sqrt[n]{x^a} \cdot \sqrt[n]{y^b}$

  Factors multiplied together inside of a radical can be broken up and written as the product of two radicals with the same index as the original. That is to say, the root of a product is equal to the product of the individual roots:

- $\sqrt[n]{\dfrac{x^a}{y^b}} = \dfrac{\sqrt[n]{x^a}}{\sqrt[n]{y^b}}$

  Just like multiplication, division problems surrounded by radicals can be broken up into separate, smaller radicals as well. So, the root of the quotient is equal to the quotient of the individual roots.

Simplifying radicals

The most common task you'll face in the study of radicals is the need to simplify radical expressions.

Example 6: Use the properties of radicals to simplify these expressions:

(a) $\sqrt{200x^2y}$

Your goal will be to break this radical into two different radicals, one containing all **perfect squares** and the other containing everything else. *Perfect squares* are quantities generated by multiplying some value by itself.

$$\sqrt{100 \cdot 2 \cdot x^2 \cdot y}$$
$$\sqrt{100x^2} \cdot \sqrt{2y}$$

Both 100 and x^2 are perfect squares (since $100 = 10 \cdot 10$ and $x^2 = x \cdot x$); the leftmost radical will be eliminated.

$$10|x|\sqrt{2y}$$

You might not have expected the absolute value signs. They are rare but necessary when you have this situation: $\sqrt[n]{x^n}$ and n is an even integer. This precaution ensures that the answer is positive, because a radical with an even index must always be positive.

(b) $\sqrt[3]{-108x^2y^8}$

Again, split up the radical, but this time put all of the **perfect cubes** (values generated by multiplying the same thing by itself three times) together:

$$\sqrt[3]{-27y^6} \cdot \sqrt[3]{4x^2y^2}$$
$$\left(-3y^2\right)\sqrt[3]{4x^2y^2}$$

There is no need to worry about absolute value signs because the index of this radical is odd.

Operations with radicals

It is a bit more complicated to add and subtract radical expressions than it is to multiply and divide them. In fact, radicals must have the same index and radicand in order to perform addition and subtraction, but that is not the case for multiplication and division.

Example 7: Simplify the following expressions:

(a) $5\sqrt{2} - 3\sqrt{8}$

While the indices are the same (they are both 2), the radicands appear different at first glance. That changes when you simplify the expression:

$$5\sqrt{2} - 3\sqrt{4}\sqrt{2}$$
$$5\sqrt{2} - 6\sqrt{2}$$

Now that they share the same radicand as well, you can combine the coefficients and get $-\sqrt{2}$.

(b) $\left(\sqrt{x}\right)\left(\sqrt[3]{x^2}\right)$

Begin by rewriting the radicals as exponential expressions:

$$x^{1/2} \cdot x^{2/3}$$

Apply Rule 1 for exponential expressions:

$$x^{1/2+2/3} = x^{3/6+4/6} = x^{7/6} = \sqrt[6]{x^7}$$

You may write your final answer in either exponential or radical form; they are equivalent.

Rationalizing expressions

Some teachers require that you **rationalize** your answers, when appropriate. This means they don't want an answer containing a radical sign in its denominator.

Example 8: Rationalize the following fraction:

$$\frac{x}{\sqrt{3}}$$

To eliminate the radical, multiply both the numerator and denominator by a value of $\sqrt{3}$. This is the equivalent of multiplying by 1, so it doesn't change the value of the fraction, and it creates a perfect square in the denominator.

$$\frac{x}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{x\sqrt{3}}{\sqrt{9}} = \frac{x\sqrt{3}}{3}$$

Polynomial Expressions

Polynomials are strings of **terms** added to or subtracted from one another. Each term is made up of numbers and variables (usually raised to whole number powers) multiplied together. For example, the polynomial

$$4x^3 - 2x^2 + x + 7$$

is made up of four terms. The **coefficient** is the numerical value preceding the variable in each term. The first term, $4x^3$, has a *coefficient* of 4, and