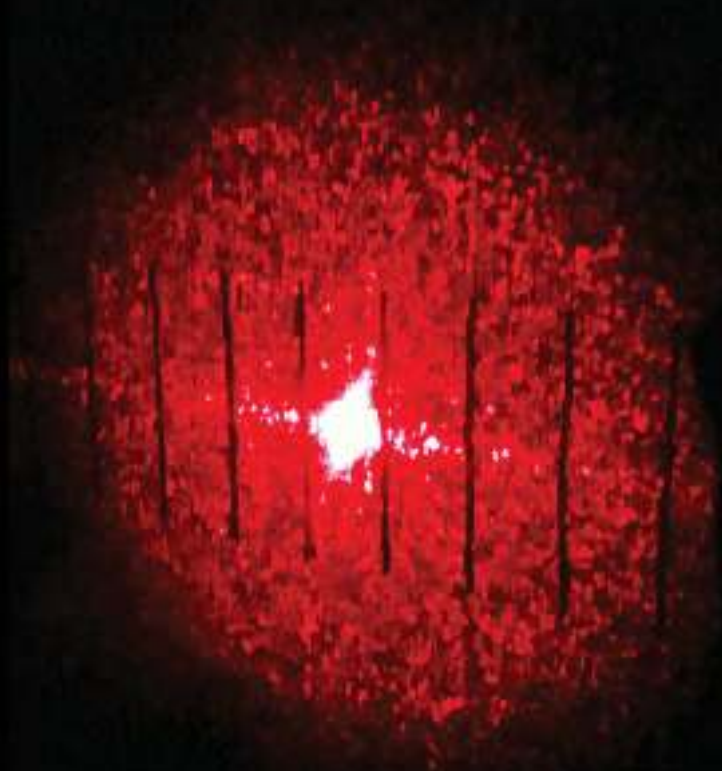


# Random Light Beams

Theory and Applications



Olga Korotkova

 CRC Press  
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Boca Raton London New York

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6000 Broken Sound Parkway NW, Suite 300  
Boca Raton, FL 33487-2742

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Version Date: 20130725

International Standard Book Number-13: 978-1-4398-1951-7 (eBook - PDF)

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## Foreword

*“..... I'd gladly be locked up in a dungeon ten fathoms below ground, if in return I could find out one thing: What is light?”*

*Galileo Galilei*

I wrote this book during my first several years of professorship at the Department of Physics of the University of Miami, FL. Being heavily involved in the research topics discussed in this text since the time of my dissertation, I recently started to realize that certain subtleties that at some point seemed to have transparent explanations could readily escape from memory. My own need for this monograph became apparent. It also was so for my group of graduate and undergraduate students as well as visiting scholars. Even if technically it is not my first book (my Ph.D. thesis has recently been published in Germany), I perceive it as being such. It is also dearer to me since while working on this text I have discovered tons of facts in areas of optics that I had thought before I knew well. Moreover, it was a remarkable revelation that namely the author of the book seems to learn more than everybody else from it. By no means is this text designed as a self-sufficient account of classical statistical optics. It is only meant to be an upgrade for a particular direction, the one relating to various phenomena associated with beam-like fields which are random in nature.

In Chapter 1 the essential mathematical and physical concepts needed for deeper understanding of the main text are reviewed. Various classes of deterministic paraxial beams are introduced in Chapter 2. Some of these beams are used in subsequent chapters as building blocks for random beams. Chapter 3 concerns random scalar beams, which were explored in depth several decades ago and discussed in detail in other books, for instance, the fundamental text *Optical Coherence and Quantum Optics* by L. Mandel and E. Wolf and *Coherent Mode Representation in Optics* by A. Ostrovsky. Hence, we will only briefly present the well-established issues relating to scalar beams and focus on the findings not yet reflected in the literature. In Chapter 4 we introduce electromagnetic random beams and point out the matters relating to their generation, propagation in free space and various media, transmission through optical systems, etc. Some of this analysis can be found in a fairly new book titled *Introduction to the Theories of Coherence and Polarization of Light* by E. Wolf. This book, however, treats a greater variety of aspects and goes deeper into details. Chapters 5 to 8 discuss some of the applications that benefit from the use of random beams. In particular, in Chapter 5 the interac-

tion of beams with deterministic optical systems is explored and the examples are given of light propagation through the human eye, laser resonators, negative phase materials, etc. Chapters 6 and 7 are concerned with propagation of random beams in random media, such as the atmosphere and ocean, as well as with optical systems operating in their presence. Finally, Chapter 8 is devoted to scattering of random beams from collections of scatterers and thin random layers, such as bio-tissue slices.

While being the sole author of this monograph I am indebted to my advisors, colleagues and students who through both short in-office discussions and profound e-mail correspondence stimulated my interest in this diverse and fast-growing field. At times entire articles of my graduate students N. Farwell, S. Sahin, and Z. Tong were used as book sections and uncountable number of times their comments and ideas were included. I am particularly grateful for the financial support of our research group provided in recent years by the US Air Force Research Office (A. Nachman, K. Miller), the US Office of Naval Research (R. Malek-Madani) and also by the Physics Department and College of Arts and Sciences at the University of Miami. The last but not least acknowledgment goes to my colleagues and co-authors of more than a hundred peer-reviewed papers including Y. Cai, G. Gbur, F. Gori, D. Zhao, J. Pu, E. Wolf, L. Andrews, R. Phillips, Y. Baykal, S. Avramov-Zamurovic, C. Nelson, E. Shchepakina, Z. Mei and many others for their perpetual guidance and shared insight into statistical optics, mathematical modeling and optical engineering.

Olga Korotkova  
January 2013



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