The MαTH βOOK

Clifford A. Pickover

From Pythagoras to the 57th Dimension,
250 Milestones in the History of Mathematics

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Books by Clifford A. Pickover

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250 Milestones in the History of Mathematics

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For Martin Gardner

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"Mathematics, rightly viewed, possesses not only truth, but supreme beauty—a beauty cold and austere, like that of sculpture."

—Bertrand Russell, *Mysticism and Logic*, 191

"Mathematics is a wonderful, mad subject, full of imagination, fantasy, and creativity that is not limited by the petty details of the physical world, but only by the strength of our inner light."

—Gregory Chaitin, "Less Proof, More Truth
New Scientist, July 28, 200

"Perhaps an angel of the Lord surveyed an endless sea of chaos, then troubled it gently with his finger. In this tiny and temporary swirl of equations, our cosmos took shape."

—Martin Gardner, *Order and Surprise*, 195

"The great equations of modern physics are a permanent part of scientific knowledge, which may outlast even the beautiful cathedrals of earlier ages."

—Steven Weinberg, in Graham Farmelo
It Must Be Beautiful, 200

HOW TO USE THIS BOOK

The 250 chronological milestones are easily viewed in the selectable table of contents that follows. Each milestone consists of a synopsis, followed by at least one image that helps to illustrate an aspect of the seminal event, publication, or concept. Occasional text in bold type points the reader to related entries. Additionally, a small “See also” section at the bottom of each entry helps weave entries together in a web of interconnectedness and may help the reader traverse the book in a playful quest for discovery.

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Introduction

The Beauty and Utility of Mathematics

> *"An intelligent observer seeing mathematicians at work might conclude that they are devotees of exotic sects, pursuers of esoteric keys to the universe."*
>
> —Philip Davis and Reuben Hersh, *The Mathematical Experienc*

Mathematics has permeated every field of scientific endeavor and plays an invaluable role in biolog physics, chemistry, economics, sociology, and engineering. Mathematics can be used to help explain the colors of a sunset or the architecture of our brains. Mathematics helps us build supersonic aircraf and roller coasters, simulate the flow of Earth's natural resources, explore subatomic quantum realities, and image faraway galaxies. Mathematics has changed the way we look at the cosmos.

In this book, I hope to give readers a taste for mathematics using few formulas, while stretching ar exercising the imagination. However, the topics in this book are not mere curiosities with little value to the average reader. In fact, reports from the U.S. Department of Education suggest that successfull completing a mathematics class in high school results in better performance at college *whatever maj* the student chooses to pursue.

The *usefulness* of mathematics allows us to build spaceships and investigate the geometry of our universe. Numbers may be our first means of communication with intelligent alien races. Some physicists have even speculated that an understanding of higher dimensions and of *topology*—the study of shapes and their interrelationships—may someday allow us to escape our universe, when it ends in either great heat or cold, and then we could call all of space-time our home.

Simultaneous discovery has often occurred in the history of mathematics. As I mention in my boo *The Möbius Strip*, in 1858 the German mathematician August Möbius (1790–1868) simultaneously and independently discovered the Möbius strip (a wonderful twisted object with just one side) along with a contemporary scholar, the German mathematician Johann Benedict Listing (1808–1882). This simultaneous discovery of the Möbius band by Möbius and Listing, just like that of calculus by English polymath Isaac Newton (1643–1727) and German mathematician Gottfried Wilhelm Leibni (1646–1716), makes me wonder why so many discoveries in science were made at the same time by people working independently. For another example, British naturalists Charles Darwin (1809–1882 and Alfred Wallace (1823–1913) both developed the theory of evolution independently and simultaneously. Similarly, Hungarian mathematician János Bolyai (1802–1860) and Russian mathematician Nikolai Lobachevsky (1793–1856) seemed to have developed hyperbolic geometry independently, and at the same time.

Most likely, such simultaneous discoveries have occurred because the time was ripe for such discoveries, given humanity's accumulated knowledge at the time the discoveries were made. Sometimes, two scientists are stimulated by reading the same preliminary research of one of their contemporaries. On the other hand, mystics have suggested that a deeper meaning exists to such coincidences. Austrian biologist Paul Kammerer (1880–1926) wrote, "We thus arrive at the image of world-mosaic or cosmic kaleidoscope, which, in spite of constant shuffling and rearrangements, also takes care of bringing like and like together." He compared events in our world to the tops of ocean waves that seem isolated and unrelated. According to his controversial theory, we notice the tops of the waves, but beneath the surface some kind of synchronistic mechanism may exist that mysterious connects events in our world and causes them to cluster.

Georges Ifrah in *The Universal History of Numbers* discusses simultaneity when writing about

Mayan mathematics:

> We therefore see yet again how people who have been widely separated in time or space have… been led to very similar if not identical results…. In some cases, the explanation for this may be found in contacts and influences between different groups of people….The true explanation lies in what we have previously referred to as the profound unity of culture: the intelligence of *Homo sapiens* is universal and its potential is remarkably uniform in all parts of the world.

Ancient people, like the Greeks, had a deep fascination with numbers. Could it be that in difficult times numbers were the only constant thing in an ever-shifting world? To the Pythagoreans, an ancie Greek sect, numbers were tangible, immutable, comfortable, eternal—more reliable than friends, less threatening than Apollo and Zeus.

Many entries in this book deal with whole numbers, or integers. The brilliant mathematician Paul Erdös (1913–1996) was fascinated by number theory—the study of integers—and he had no trouble posing problems, using integers, that were often simple to state but notoriously difficult to solve. Erdös believed that if one can state a problem in mathematics that is unsolved for more than a centur then it is a problem in number theory.

Many aspects of the universe can be expressed by whole numbers. Numerical patterns describe the arrangement of florets in a daisy, the reproduction of rabbits, the orbit of the planets, the harmonies o music, and the relationships between elements in the periodic table. Leopold Kronecker (1823–1891) a German algebraist and number theorist, once said, “The integers came from God and all else was man-made.” His implication was that the primary source of all mathematics is the integers.

Since the time of Pythagoras, the role of integer ratios in musical scales has been widely appreciated. More important, integers have been crucial in the evolution of humanity’s scientific understanding. For example, French chemist Antoine Lavoisier (1743–1794) discovered that chemica compounds are composed of fixed proportions of elements corresponding to the ratios of small integers. This was very strong evidence for the existence of atoms. In 1925, certain integer relations between the wavelengths of spectral lines emitted by excited atoms gave early clues to the structure o atoms. The near-integer ratios of atomic weights were evidence that the atomic nucleus is made up o an integer number of similar nucleons (protons and neutrons). The deviations from integer ratios led to the discovery of elemental isotopes (variants with nearly identical chemical behavior but with different numbers of neutrons).

Small divergences in the atomic masses of pure isotopes from exact integers confirmed Einstein’s famous equation $E = mc^2$ and also the possibility of atomic bombs. Integers are everywhere in atomi physics. Integer relations are fundamental strands in the mathematical weave—or as German mathematician Carl Friedrich Gauss (1777–1855) said, “Mathematics is the queen of sciences—and number theory is the queen of mathematics.”

Our mathematical description of the universe grows forever, but our brains and language skills remain entrenched. New kinds of mathematics are being discovered or created all the time, but we need fresh ways to think and to understand. For example, in the last few years, mathematical proofs have been offered for famous problems in the history of mathematics, but the arguments have been fa too long and complicated for experts to be certain they are correct. Mathematician Thomas Hales had to wait *five years* before expert reviewers of his geometry paper—submitted to the journal *Annals of Mathematics*—finally decided that they could find no errors and that the journal should publish Hales’s proof, but only with the disclaimer saying they were not certain it was right! Moreover, mathematicians like Keith Devlin have admitted in the *New York Times* that “the story of mathemati has reached a stage of such abstraction that many of its frontier problems cannot even be understood by the experts.” If experts have such trouble, one can easily see the challenge of conveying this kind of ~~information to a general audience. We do the best we can. Mathematicians can construct theories~~ and perform computations, but they may not be sufficiently able to fully comprehend, explain, or communicate these ideas.

A physics analogy is relevant here. When Werner Heisenberg worried that human beings might never truly understand atoms, Niels Bohr was a bit more optimistic. He replied in the early 1920s, “I think we may yet be able to do so, but in the process we may have to learn what the word *understanding* really means.” Today, we use computers to help us reason beyond the limitations of ou own intuition. In fact, experiments with computers are leading mathematicians to discoveries and insights never dreamed of before the ubiquity of these devices. Computers and computer graphics allow mathematicians to discover results long before they can prove them formally and open entirely new fields of mathematics. Even simple computer tools like spreadsheets give modern mathematicians power that Gauss, Leonhard Euler, and Newton would have lusted after. As just one example, in the late 1990s, computer programs designed by David Bailey and Helaman Ferguson helped produce new formulas that related pi to log 5 and two other constants. As Erica Klarreich reports in *Science News*, once the computer had produced the formula, proving that it was correct wa extremely easy. Often, simply *knowing* the answer is the largest hurdle to overcome when formulatin a proof.

Mathematical theories have sometimes been used to predict phenomena that were not confirmed until years later. For example, Maxwell’s equations, named after physicist James Clerk Maxwell, predicted radio waves. Einstein’s field equations suggested that gravity would bend light and that the universe is expanding. Physicist Paul Dirac once noted that the abstract mathematics we study now gives us a glimpse of physics in the future. In fact, his equations predicted the existence of antimatte which was subsequently discovered. Similarly, mathematician Nikolai Lobachevsky said that “there no branch of mathematics, however abstract, which may not someday be applied to the phenomena o the real world.”

In this book, you will encounter various interesting geometries that have been thought to hold the keys to the universe. Galileo Galilei (1564–1642) suggested that “Nature’s great book is written in mathematical symbols.” Johannes Kepler (1571–1630) modeled the solar system with Platonic solids such as the dodecahedron. In the 1960s, physicist Eugene Wigner (1902–1995) was impressed with th “unreasonable effectiveness of mathematics in the natural sciences.” Large Lie groups, like E_8—which is discussed in the entry “The Quest for Lie Group E_8 (2007)”—may someday help us create a unified theory of physics. In 2007, Swedish American cosmologist Max Tegmark published both scientific and popular articles on the mathematical universe hypothesis, which states that our physica reality is a mathematical structure—in other words, our universe is not just *described* by mathematic —it *is* mathematics.

Book Organization and Purpose

> *“At every major step, physics has required, and frequently stimulated, the introduction of new mathematical tools and concepts. Our present understanding of the laws of physics, with their extreme precision and universality, is only possible in mathematical terms.”*
>
> —Sir Michael Atiyah, “Pulling the Strings,” *Natu*

One common characteristic of mathematicians is a passion for completeness—an urge to return to first principles to explain their works. As a result, readers of mathematical texts must often wade through pages of background before getting to the essential findings. To avoid this problem, each entry in this book is short, at most only a few paragraphs in length. This format allows readers to jum ~~right in to ponder a subject, without having to sort through a lot of verbiage. Want to know about~~ infinity? Turn to the entries “Cantor’s Transfinite Numbers” (1874) or “Hilbert’s Grand Hotel” (1925), and you’ll have a quick mental workout. Interested in the first commercially successful portable mechanical calculator, developed by a prisoner in a Nazi concentration camp? Turn to “Curt Calculator” (1948) for a brief introduction.

Wonder how an amusing-sounding theorem may one day help scientists form nanowires for electronics devices? Then browse through the book and read the “Hairy Ball Theorem” (1912) entry. Why did the Nazis compel the president of the Polish Mathematical Society to feed his own blood to lice? Why was the first female mathematician murdered? Is it really possible to turn a sphere inside out? Who was the “Number Pope”? When did humans tie their first knots? Why don’t we use Roman numerals anymore? Who was the earliest named individual in the history of mathematics? Can a surface have only one side? We’ll tackle these and other thought-provoking questions in the pages th follow.

Of course, my approach has some disadvantages. In just a few paragraphs, I can’t go into any depth on a subject. However, I provide suggestions for further reading in the “Notes and Further Reading” section. While I sometimes list primary sources, I have often explicitly listed excellent secondary references that readers can frequently obtain more easily than older primary sources. Readers interested in pursuing any subject can use the references as a useful starting point.

My goal in writing *The Math Book* is to provide a wide audience with a brief guide to important mathematical ideas and thinkers, with entries short enough to digest in a few minutes. Most entries a ones that interested me personally. Alas, not all of the great mathematical milestones are included in this book in order to prevent the book from growing too large. Thus, in celebrating the wonders of mathematics in this short volume, I have been forced to omit many important mathematical marvels. Nevertheless, I believe that I have included a majority of those with historical significance and that have had a strong influence on mathematics, society, or human thought. Some entries are eminently practical, involving topics that range from slide rules and other calculating devices to geodesic dome and the invention of zero. Occasionally, I include several lighter moments, which were nonetheless significant, such as the rise of the Rubik’s Cube puzzle or the solving of the Bed Sheet Problem. Sometimes, snippets of information are repeated so that each entry can be read on its own. Occasiona text in boldface type points the reader to related entries. Additionally, a small “See also” section at th bottom of each entry helps weave entries together in a web of interconnectedness and may help the reader traverse the book in a playful quest for discovery.

The Math Book reflects my own intellectual shortcomings, and while I try to study as many areas o science and mathematics as I can, it is difficult to become fluent in all aspects, and this book clearly indicates my own personal interests, strengths, and weaknesses. I am responsible for the choice of pivotal entries included in this book and, of course, for any errors and infelicities. This is not a comprehensive or scholarly dissertation, but rather it is intended as recreational reading for students of science and mathematics and interested laypeople. I welcome feedback and suggestions for improvement from readers, as I consider this an ongoing project and a labor of love.

This book is organized chronologically, according to the year of a mathematical milestone or finding. In some cases, the literature may report slightly different dates for the milestone because some sources give the publication date as the discovery date of a finding, while other sources give th actual date that a mathematical principle was discovered, regardless of the fact that the publication date is sometimes a year or more later. If I was uncertain of a precise earlier date of discovery, I ofte used the publication date.

Dating of entries can also be a question of judgment when more than one individual made a contribution. Often, I have used the earliest date where appropriate, but sometimes I have surveyed coll~~eagues and decided to use the date when a concept gained particular prominence. For exampl~~e, consider the Gray code, which is used to facilitate error correction in digital communications, such a in TV signal transmission, and to make transmission systems less susceptible to noise. This code wa named after Frank Gray, a physicist at Bell Telephone Laboratories in the 1950s and 1960s. During this time, these kinds of codes gained particular prominence, partly due to his patent filed in 1947 an the rise of modern communications. The Gray code entry is thus dated as 1947, although it might als have been dated much earlier, because the roots of the idea go back to Émile Baudot (1845–1903), th French pioneer of the telegraph. In any case, I have attempted to give readers a feel for the span of possible dates in each entry or in the "Notes and Further Reading" section.

Scholars sometimes have disputes with respect to the person to whom a discovery is traditionally attributed. For example, author Heinrich Dörrie cites four scholars who do not believe that a particul version of Archimedes' cattle problem is due to Archimedes, but he also cites four authors who believe that the problem *should* be attributed to Archimedes. Scholars also dispute the authorship of Aristotle's wheel paradox. Where possible, I mention such disputes either in the main text or the "Notes and Further Reading" section.

You will notice that a significant number of milestones have been achieved in just the last few decades. As just one example, in 2007, researchers finally "solved" the game of checkers, proving th if an opponent plays perfectly, the game ends in draw. As already mentioned, part of the rapid recent progress in mathematics is due to the use of the computer as a tool for mathematical experiments. Fo the checkers solution, the analysis actually began in 1989 and required dozens of computers for the complete solution. The game has roughly 500 billion billion possible positions.

Sometimes, science reporters or famous researchers are quoted in the main entries, but purely for brevity I don't list the source of the quote or the author's full credentials in the entry. I apologize in advance for this occasional compact approach; however, references in the back of the book should help to make the author's identity clearer.

Even the naming of a theorem can be a tricky business. For example, mathematician Keith Devlin writes in his 2005 column for the Mathematical Association of America:

> Most mathematicians prove many theorems in their lives, and the process whereby their name gets attached to one of them is very haphazard. For instance, Euler, Gauss, and Fermat each proved hundreds of theorems, many of them important ones, and yet their names are attached to just a few of them. Sometimes theorems acquire names that are incorrect. Most famously, perhaps, Fermat almost certainly did not prove "Fermat's Last Theorem"; rather, that name was attached by someone else, after his death, to a conjecture the French mathematician had scribbled in the margi of a textbook. And Pythagoras's theorem was known long before Pythagoras came onto the scene.

In closing, let us note that mathematical discoveries provide a framework in which to explore the nature of reality, and mathematical tools allow scientists to make predictions about the universe; thu the discoveries in this book are among humanity's greatest achievements.

At first glance, this book may seem like a long catalogue of isolated concepts and people with littl connection between them. But as you read, I think you'll begin to see many linkages. Obviously, the final goal of scientists and mathematicians is not simply the accumulation of facts and lists of formulas, but rather they seek to understand the patterns, organizing principles, and relationships between these facts to form theorems and entirely new branches of human thought. For me, mathematics cultivates a perpetual state of wonder about the nature of mind, the limits of thoughts, and our place in this vast cosmos.

Our brains, which evolved to make us run from lions on the African savanna, may not be con~~structed to penetrate the infinite veil of reality. We may need mathematics, science, compute~~rs, brain augmentation, and even literature, art, and poetry to help us tear away the veils. For those of yo who are about to embark on reading the *The Math Book* from cover to cover, look for the connections gaze in awe at the evolution of ideas, and sail on the shoreless sea of imagination.

Acknowledgments

I thank Teja Krašek, Dennis Gordon, Nick Hobson, Pete Barnes, and Mark Nandor for their comment and suggestions. I would also like to especially acknowledge Meredith Hale, my editor for this book, as well as Jos Leys, Teja Krašek, and Paul Nylander for allowing me to include their mathematically inspired artworks.

While researching the milestones and pivotal moments presented in this book, I studied a wide array of wonderful reference works and Web sites, many of which are listed in the “Notes and Furthe Reading” section toward the end of the book. These references include “The MacTutor History of Mathematics Archive” (*www-history.mcs.st-and.ac.uk*), “Wikipedia: The Free Encyclopedia” (*en.wikipedia.org*), “MathWorld” (*mathworld.wolfram.com*), Jan Gullberg’s *Mathematics: From the Birth of Numbers*, David Darling’s *The Universal Book of Mathematics*, Ivars Peterson’s “Math Trek Archives” (*www.maa.org/mathland/ mathland_archives.html*), Martin Gardner’s *Mathematical Games* (a CDROM made available from The Mathematical Association of America), and some of my own books such as *A Passion for Mathematics*.

Ant Odometer

c. 150 Million B.C.

Ants are social insects that evolved from vespoid wasps in the mid-Cretaceous period, about 150 million years ago. After the rise of flowering plants, about 100 million years ago, ants diversified int numerous species.

The Saharan desert ant, *Cataglyphis fortis*, travels immense distances over sandy terrain, often completely devoid of landmarks, as it searches for food. These creatures are able to return to their ne using a direct route rather than by retracing their outbound path. Not only do they judge directions, using light from the sky for orientation, but they also appear to have a built-in "computer" that functions like a pedometer that counts their steps and allows them to measure exact distances. An ant may travel as far as 160 feet (about 50 meters) until it encounters a dead insect, whereupon it tears a piece to carry directly back to its nest, accessed via a hole often less than a millimeter in diameter.

By manipulating the leg lengths of ants to give them longer and shorter strides, a research team of German and Swiss scientists discovered that the ants "count" steps to judge distance. For example, after ants had reached their destination, the legs were lengthened by adding stilts or shortened by partial amputation. The researchers then returned the ants so that the ants could start on their journey back to the nest. Ants with the stilts traveled too far and passed the nest entrance, while those with th amputated legs did not reach it. However, if the ants *started* their journey from their nest with the modified legs, they were able to compute the appropriate distances. This suggests that stride length is the crucial factor. Moreover, the highly sophisticated computer in the ant's brain enables the ant to compute a quantity related to the horizontal projection of its path so that it does not become lost even if the sandy landscape develops hills and valleys during its journey.

SEE ALSO Primates Count (c. 30 Million B.C.) and Cicada-Generated Prime Numbers (c. 1 Million B.C.).

Saharan desert ants may have built-in “pedometers” that count steps and allow the ants to measure exact distances. Ants with stilts glued to their legs (shown in red) travel too far and pass their nest entrance, suggesting that stride length is important for distance determination.

Primates Count

c. 30 Million B.C.

Around 60 million years ago, small, lemur-like primates had evolved in many areas of the world, and 30 million years ago, primates with monkeylike characteristics existed. Could such creatures count? The meaning of *counting* by animals is a highly contentious issue among animal behavior experts. However, many scholars suggest that animals have some sense of number. H. Kalmus writes in his *Nature* article “Animals as Mathematicians”:

> There is now little doubt that some animals such as squirrels or parrots can be trained to count…. Counting faculties have been reported in squirrels, rats, and for pollinating insects. Some of these animals and others can distinguish numbers in otherwise similar visual patterns, while others can b trained to recognize and even to reproduce sequences of acoustic signals. A few can even be traine to tap out the numbers of elements (dots) in a visual pattern….The lack of the spoken numeral and the written symbol makes many people reluctant to accept animals as mathematicians.

Rats have been shown to “count” by performing an activity the correct number of times in exchang for a reward. Chimpanzees can press numbers on a computer that match numbers of bananas in a box Testsuro Matsuzawa of the Primate Research Institute at Kyoto University in Japan taught a chimpanzee to identify numbers from 1 to 6 by pressing the appropriate computer key when she was shown a certain number of objects on the computer screen.

Michael Beran, a research scientist at Georgia State University in Atlanta, Georgia, trained chimps to use a computer screen and joystick. The screen flashed a numeral and then a series of dots, and the chimps had to match the two. One chimp learned numerals 1 to 7, while another managed to count to 6. When the chimps were tested again after a gap of three years, both chimps were able to match numbers, but with double the error rate.

SEE ALSO Ant Odometer (c. 150 Million B.C.) and Ishango Bone (c. 18,000 B.C.).

Primates appear to have some sense of number, and the higher primates can be taught to identify numbers from 1 to 6 by pressing the appropriate computer key when shown a certain number of objects.

Cicada-Generated Prime Numbers

c. 1 Million B.C.

Cicadas are winged insects that evolved around 1.8 million years ago during the Pleistocene epoch, when glaciers advanced and retreated across North America. Cicadas of the genus *Magicicada* spend most of their lives below the ground, feeding on the juices of plant roots, and then emerge, mate, and die quickly. These creatures display a startling behavior: Their emergence is synchronized with periods of years that are usually the prime numbers 13 and 17. (A prime number is an integer such as 11, 13, and 17 that has only two integer divisors: 1 and itself.) During the spring of their 13th or 17th year, these periodical cicadas construct an exit tunnel. Sometimes more than 1.5 million individuals emerge in a single acre; this abundance of bodies may have survival value as they overwhelm predators such as birds that cannot possibly eat them all at once.

Some researchers have speculated that the evolution of prime-number life cycles occurred so that the creatures increased their chances of evading shorter-lived predators and parasites. For example, if these cicadas had 12-year life cycles, all predators with life cycles of 2, 3, 4, or 6 years might more easily find the insects. Mario Markus of the Max Planck Institute for Molecular Physiology in Dortmund, Germany, and his coworkers discovered that these kinds of prime-number cycles arise naturally from evolutionary mathematical models of interactions between predator and prey. In order to experiment, they first assigned random life-cycle durations to their computer-simulated populations. After some time, a sequence of mutations always locked the synthetic cicadas into a stable prime-number cycle.

Of course, this research is still in its infancy and many questions remain. What is special about 13 and 17? What predators or parasites have actually existed to drive the cicadas to these periods? Also, a mystery remains as to why, of the 1,500 cicada species worldwide, only a small number of the genus *Magicicada* are known to be periodical.

SEE ALSO Ant Odometer (c. 150 Million B.C.), Ishango Bone (c. 18,000 B.C.), Sieve of Eratosthenes (240 B.C.), Goldbach Conjecture (1742), Constructing a Regular Heptadecagon (1796), Gauss's *Disquisitiones Arithmeticae* (1801), Proof of the Prime Number Theorem (1896), Brun's Constant (1919), Gilbreath's Conjecture (1958), Sierpinski Numbers (1960), Ulam Spiral (1963), Erdös and Extreme Collaboration (1971), and Andrica's Conjecture (1985).