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Introduction

Quantitative methods for evaluating price movement and making trading decisions have become a dominant part of market analysis. At one time, the only acceptable manner of trading was by understanding the factors that make prices move, and determining the extent or potential of future movement. The market now supports dozens of major funds and managed programs, which account for a sizable part of futures market open interest and operate primarily by decisions based on "technical analysis." selection, which can require sorting through thousands of individual world equities each day, has become a problem in data reduction-finding specific patterns that offer the best expectations of profit. Many commercial participants in the markets, who once restricted research to supply and demand, or institutions once only interested in earnings and debt, now include various technical methods for the purpose of--timing--or confirming price direction.

In many ways, there is no conflict between fundamental and technical analysis. The decisions that result from economic or policy changes are far-reaching: these actions may cause a long-term change in the direction of prices and may not be reflected immediately. Actions based on long-term forecasts may involve considerable risk and often can be an ineffective way to manage a position. Integrated with a technical method of known risk, which determines price trends over shorter intervals, investors at all levels have gained practical solutions to their trading problems.

Leverage in the futures markets has a strong influence on the methods of trading. With margin deposits ranging from 5 to 10% of the contract value (the balance does not have to be borrowed as in stocks), a small movement in the underlying price can result in large profits and losses based on the invested margin. Because high leverage is available, it is nearly always used. Methods of analysis will therefore concentrate on short-term price fluctuations and trends, in which the profit potential is reduced, so that the risk is often smaller than the required margin. Futures market systems can be characterized as emphasizing price moves of less than 20% of the contract value. Trading requires conservation of capital, and the management of investment risk becomes essential.

Even with the distinction forced by high leverage, many of the basic systems covered in this book were first used in the stock market. Compared with securities, the relatively small number of futures markets offer great diversification and liquidity. The relative lack of liquidity in a single stock lends itself to index analysis, whereas the -commodin- index, now tradeable as the CRB index, has never become very popular.

TECHNICAL VERSUS FUNDAMENTAL

Two basic approaches to trading futures are the same as in trading equities: fundamental and technical analysis. In futures, a fundamental study may be a composite of supply-and-demand elements: statistical reports on production, expected use, political ramifications, labor influences, price support programs, industrial development-everything that makes prices what they are. The result of a fundamental analysis is a price forecast, a prediction of where prices will be at some time in the future.

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Technical analysis is a study of patterns and movement. Its elements are normally limited to price, volume, and open interest. It is considered to be the study of the market itself. The results of technical analysis may be a short- or long-term forecast based on recurring patterns; however, technical methods often limit their goals to the statement that today's prices are moving up or down. Some systems will go as far as saying the direction is indeterminate.

Due to the rapid growth of computers, technical systems now use tools previously reserved for fundamental analysis. Regression and cycle (seasonal) analysis are built into most spreadsheet programs and allow these more complex studies, which were once reserved for serious fundamental analysts, to be performed by everyone. Because they are computerized, many technicians now consider them in their own domain. There will always be purists on either side, rigid fundamentalists and technicians, but a great number of professionals combine the two techniques. This book draws on some of the more popular, automated fundamental trading approaches.

One advantage of technical analysis is that it is completely self-contained. The accuracy of the data is certain. One of the first great advocates of price analysis, Charles Dow, said:

The market reflects all the jobber knows about the condition of the textile trade; all the banker knows about the money market; all that the best-informed president knows of his own business, together with his knowledge of all other businesses; it sees the general condition of transportation in a way that the president of no single railroad can ever see; it is better informed on crops than the farmer or even the Department of Agriculture. In fact, the market reduces to a bloodless verdict all

knowledge bearing on finance both domestic and foreign.

Much of the price movement reflected in commodity cash and futures markets is anticipatory; the expectations of the effects of economic developments. It is subject to change without notice. For example, a hurricane bound for the Philippines will send sugar prices higher, but if the storm turns off course, prices will drop back to prior levels. Major scheduled crop reports cause a multitude of professional guessing, which may correctly or incorrectly move prices just before the actual report is released. By the time the public is ready to act, the news is already reflected in the price.

PROFESSIONAL AND AMATEUR

Beginning traders often find a system or technique that seems extremely simple and convenient to follow, one that they think has been overlooked by the professionals. Sometimes they are right, but most often that method doesn't work. Reasons for not using a technique could be the inability to get a good execution, the risk/reward ratio, or the number of consecutive losses that occur. Speculation is a difficult business, not one to be taken casually. As Wyckoff said, "Most men make money in their own business and lose it in some other fellow's."

To compete with a professional speculator, you must be more accurate in anticipating the next move or in predicting prices from current news—not the article printed in today's newspaper ("Government Buys Beef for School Lunch Program"), which was discounted weeks ago, and not the one on the wire service ("15% Fewer Soybeans and 10% More Fishmeal") which went into the market two days ago. You must act on news that has not yet been printed. To anticipate changes, you must draw a single conclusion for the many contingencies possible from fundamental data, or

1. Recognize recurring patterns in price movement and determine the most likely results of such patterns.
2. Determine the trend of the market by isolating the basic direction of prices over a selected time interval.

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The bar chart, discussed in Chapter 9 ("Charting"), is the simplest representation of the market. These patterns are the same as those recognized by Livermore on the ticker tape. Because they are interpretive, more precise methods such as point-and-figure charting are also used, which add a level of exactness to charting. Point-and-figure charts are popular because they offer specific trading rules and show formations similar to both bar charting and ticker-tape trading.

Mathematical modeling, using traditional regression or discrete analysis, has become a popular technique for anticipating price direction. Most modeling methods are modifications of developments in econometrics, basic probability; and statistical theory. They are precise because they are based entirely on numerical data.

The proper assessment of the price trend is critical to most commodity trading systems. Countertrend trading is just as dependent on knowing the trend as a trend-following technique. Large sections of this book are devoted to the various ways to isolate the trend, although it would be an injustice to leave the reader with the idea that a price trend is a universally accepted concept. There have been many studies published claiming that trends, with respect to price movement, do not exist. The most authoritative papers on this topic are collected in Cootner, *The Random Character of Stock Market Prices* (MIT Press) more recent and readable discussions can often be found in *The Financial Analysts Journal*, an excellent resource.

Personal financial management has gained an enormous number of tools during this period of computerized expansion. The major spreadsheet providers include linear regression and correlation analysis; there is inexpensive software to perform spectral analysis and apply advanced statistical techniques; and development software, such as TradeStation and MetaStock, have provided trading platforms and greatly reduced the effort needed to program your ideas. The professional maintains the advantage of having all of their time to concentrate on the investment problems; however, the nonprofessional is no longer at a disadvantage because of the tools.

RANDOM WALK

It has been the position of many fundamental and economic analysis advocates that there is no sequential correlation between the direction of price movement from one day to the next. Their position is that prices will seek a level that will balance the supply-demand factors, but that this level will be reached in an unpredictable manner as prices move in an irregular response to the latest available information or news release.

If the random walk theory is correct, many well-defined trading methods based on mathematics and pattern recognition will fail. The problem is not a simple one, but one that should be resolved by each system developer, because it will influence the type of systematic approaches that will be studied. The strongest argument against the

random movement supporters is one of price anticipation. One can argue academically that all participants (the market) know exactly where prices should move following the release of news. However practical or unlikely this is, it is not as important as market movement based on anticipation of further movement. For example, if the prime rate was raised twice in two months, would you expect it to be increased in the third month? Do you think that others will have mixed opinions, or that they assess the likelihood of another increase at different levels (i.e., one might see a 25% chance of an increase and another see a 60% chance). Unless the whole market view expectations the same way, then the price will move to reflect the majority opinion. As news alters that opinion the market will fluctuate. Is this random movement? No. Can this appear similar to random movement? Yes.

Excluding anticipation, the apparent random movement of prices depends on both the time interval and the frequency of data used. When a long time span is used, from 1 to

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20 years, and the data averaged to increase the smoothing process, the trending characteristics will change, along with seasonal and cyclic variations. Technical methods, such as moving averages, are often used to isolate these price characteristics. The averaging of data into quarterly prices smooths out the irregular daily movements and results in noticeably positive correlations between successive prices. The use of daily data over a long time interval introduces noise and obscures uniform patterns.

In the long run, most futures prices find a level of equilibrium (with the exception of the stock index, which has had an upward bias) and, over some time period, show the characteristics of being mean reverting (returning to a local average price); however, short-term price movement can be very different from a random series of numbers. It often contains two unique properties: exceptionally long runs of price in a single direction, and asymmetry, the unequal size of moves in different directions. These are the qualities that allow traders to profit. Although the long-term trends that reflect economic policy, easily seen in the quarterly data, are not of great interest to futures traders, short-term price movements-caused by anticipation rather than actual events, extreme volatility, prices that are seen as far from value, countertrend systems that rely on mean reversion, and those that attempt to capture trends of less duration-have been successful.

It is always worthwhile to understand the theoretical aspects of price movement, because it does paint a picture of the way prices move. Many traders have been challenged by trying to identify the difference between an actual daily price chart and one created by a random number generator. There are differences, but they will seem more subtle than you would expect. The ability to identify those differences is the same as finding a way to profit from actual price movements. A trading program seeks to find ways to operate within the theoretical framework, looking for exceptions, selecting a different time frame and capture profits-and all without ignoring the fact that the theory accounts for most of the price movements.

BACKGROUND MATERIAL

The contents of this book assume an understanding of speculative markets, particularly the futures markets. Ideally the reader should have read one or more of the available trading guides, and understand the workings of a buy or sell order and the specifications of contracts. Experience in actual trading would be helpful. A professional trader, a broker, or a purchasing agent will already possess all the qualifications necessary. A farmer or rancher with some hedging experience will be well qualified to understand the risks involved. So is any investor who manages his or her own stock portfolio.

Literature on markets and trading systems has greatly expanded in the 11 years since the last edition of this book. During that time the most comprehensive and excellent work has been Jack Schwager's two-volume set, *Schwager on Futures* (Wiley, 1995), which includes one volume on fundamental analysis and the other on technical analysis. John Murphey's *Technical Analysis of the Futures Markets* (New York Institute of Finance, 1986) and *Intermarket Technical Analysis* (Wiley, 1991) are highly recommended. Ralph Vince published a popular work, *Portfolio Management Formulas* (Wiley, 1990), and there is Peter L. Bernstein's *The Portable MBA in Investment* (Wiley, 1995), which again provides valuable background material in readable form. There have been quite a few books on specific systems and some on the development of computerized trading methods. The one comprehensive book of studies that stands out is *The Encyclopedia of Technical Market Indicators* by Robert W. Colby and Thomas A. Meyers (Dow Jones-Irwin, 1988), which offers an intelligent description of the calculation and trading performance of most market indicators oriented toward equities traders. Comparing the results of different indicators, side by side, can give you valuable insight into the practical differences in these techniques.

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The basic reference book for general contract information has always been the *Commodity Trading Manual*

(Chicago Board of Trade), but each year Futures magazine publishes a Reference Guide, which gives the current futures and options markets traded around the world. No doubt, all of this information will be available through Internet. For beginning or reviewing the basics, there is Todd Lofton's *Getting Started in Futures* (Wiley, 1989); Little and Rhodes, *Understanding Wall Street, Third Edition* (McGraw-Hill, 1991); and *The Stock Market, 6th Edition* by Teweles, Bradley, and Teweles (Wiley, 1992). The introductory material is not repeated here.

A good understanding of the most popular charting method requires reading the classic by Edwards and Magee, *Technical Analysis of Stock Trends* (John Magee), a comprehensive study of bar charting. Writings on other technical methods are more difficult to find. The magazine *Technical Analysis of Stocks & Commodities* stands out as the best source of regular information; Futures magazine has fewer technical articles, but many of value and many other commodity books express only a specific technical approach. Current analysis of many market phenomena and relationships can be found in *The Financial Analysts Journal*.

On general market lore, and to provide motivation when trading is not going as well as expected, the one book that stands out is Lefevre's *Reminiscences of a Stock Operator* (originally published by Doran, reprinted by Wiley in 1994). Wyckoff mixes humor and philosophy in most of his books, but *Wall Street Ventures and Adventures Through Forty Years* (Harper & Brothers) may be of general interest. More recently, Jack Schwager's *Market Wizards* (New York Institute of Finance, 1989) has been very popular.

A reader with a good background in high school mathematics can follow most of this book, except in its more complex parts. An elementary course in statistics is ideal, but a knowledge of the type of probability found in Thorp's *Beat the Dealer* (Vintage) is adequate. Fortunately, computer spreadsheet programs, such as Excel and Quattro, allow anyone to use statistical techniques immediately, and most of the formulas in this book are presented in such a way that they can be easily adapted to spreadsheets. Having a computer with trading software (such as Omega's SuperCharts, MetaStock, or any number of products), or having a data feed (such as Telerate or CQG), which offers technical studies, you are well equipped to continue.

RESEARCH SKILLS

Before starting, a few guidelines may help make the task easier. They have been set down to help those who will use this book to develop a trading system.

1. Know what you want to do. Base your trading on a solid theory or observation, and keep it in focus throughout development and testing. This is called the underlying premise of your program.
 2. State your hypothesis or question in its simplest form. The more complex it is, the more difficult it will be to evaluate the answer.
 3. Do not assume anything. Many projects fail on basic assumptions that were incorrect.
 4. Do the simplest things first. Do not combine systems before each element of each system is proven to work independently.
 5. Build one step at a time. Go on to the next step only after the previous ones have been tested successfully. If you start with too many complex steps and fail, you will have to simplify to find out what went wrong.
 6. Be careful of errors of omission. The most difficult part of research is identifying the components to be selected and tested. Simply because all the questions asked were satisfactorily answered does not mean that all the right questions were asked. The most important may be missing.
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7. Do not take shortcuts. It is sometimes convenient to use the work of others to speed up the research. Check their work carefully; do not use it if it cannot be verified. Check your spreadsheet calculations manually. Remember that your answer is only as good as its weakest point.
 8. Start at the end. Define your goal and work backward to find the required input. In this manner, you only work with information relevant to the results; otherwise, you might spend a great deal of time on irrelevant items.

OBJECTIVES OF THIS BOOK

This book is intended to give you a complete understanding of the tools and techniques needed to develop or

choose a trading program that has a good chance of being successful. Execution skill and market psychology are not considered, but only the development of a system that has been carefully thought out and tested. This itself is an achievement of no small magnitude.

Not everything can be covered in a single book; therefore, some guidelines were needed to control the material included here. Most important are techniques that are common to most markets, such as trend and countertrend techniques, indicators, and testing methods. Popular analytic techniques, such as charting, are only covered to the degree that various patterns can be used in a computerized program to help identify support and resistance, channels, and so forth. There has been no attempt to provide a comprehensive text on charting. Various formations may offer very realistic profit objectives or provide reliable entry filters, even though they are not included.

Some popular areas, such as options, are not covered at all. There are many good books on options strategies, and to include them here would be a duplication of effort. Also, those strategies that use statistics, such as price/earnings ratios, specific to equities, have not been included, although indicators that use volume, even the number of advancing and declining issues, you will find in the section on volume because they fit into a bigger picture. This remains a book on trading futures markets, yet it recognizes that many methods can be used elsewhere.

This book will not attempt to prove that one system is better than another, because it is not possible to know what will happen in the future. It will try to evaluate the conditions under which certain methods are likely to do better and situations that will be harmful to specific approaches. Most helpful should be the groupings of systems and techniques, which allow a comparison of features and possible results. Seeing how analysts have modified existing ideas can help you decide how to proceed, and why you might choose one path over another. By seeing a more complete picture, it is hoped that common sense will prevail, rather than computing power.

PROFILE OF A TRADING SYSTEM

There are quite a few steps to be considered when developing a trading program. Some of these are simply choices in style that must be made, while others are essential to the success of the results. They have been listed here and discussed briefly as items to bear in mind as you continue the process of creating a trading system.

Changing Markets and System Longevity

Markets are not static. They evolve because the world changes. Among those items that have changed during the past 10 years are the market participants, the tools used to watch the market, the tools used to develop trading models, the economies of countries such as Japan, the union of European countries, the globalization of markets, and the risk of participation. Under this changing situation, a trading system that works today might not work

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far into the future. We must carefully consider how each feature of a trading program is affected by change and try to create a method that is as robust as possible to increase its longevity.

The Choice of Data

System decisions are limited by the data used in the analysis. Although price and volume for the specific market may be the definitive criteria, there is a multitude of other valid statistical information that might also be used. Some of this data is easily included, such as price data from related markets; other statistical data, including the U.S. economic reports and weekly energy inventories, may add a level of robustness to the results but are less convenient to obtain.

Diversification

Not all traders are interested in diversification, which tends to reduce returns at the same time that it limits risk. Concentrating all of your resources on a single market that you understand may produce a specialized approach and much better results than using a more general technique over more markets. Diversification may be gained by trading more than one method in addition to a broad set of markets, provided the programs are unique in style. Proper diversification reduces risk more than returns.

Time Frame

The time frame of the data impacts both the type of system and the nature of the results. Using 5minute bars

introduces considerable noise to your program, making it difficult to find the trend, while using only weekly data puts so much emphasis on the trend such that your trading style is already determined. A shorter time may guarantee faster response to price changes, but it does not assure better results. Each technique must be applied properly to the right data and time frame.

Choosing a Method of Analysis

Some methods of analyzing the market are more complex than others. This in itself has no bearing on the final success. All good trading methods begin with a sound premise. You must first know what you are trying to extract from the market before you select a technique. If you want to capitalize on long interest rate trends or on the result of government policy, then a weekly moving average or trend system will be the place to start. If you see false breakouts whenever price penetrates the high of the day in the second half of the trading session, you'll want to look at a momentum indicator based on 5-, 10-, or 15-minute data. First the idea, then the tool.

Trade Selection

Although a trading system produces signals regularly, it is not necessary to enter all of them. Selecting one over another can be done by a method of filtering- This can vary from a confirmation by another technique or system, a limitation on the amount of risk that can be accepted on any one trade, the use of outside information, or the current volume. Many of these add a touch of reality to an automated process. You may find, however, that too many filters result in no trading.

Testing

There has been a lot of emphasis on testing, and there is a complete discussion in this book; however, testing is most important to confirm, or validate, your ideas. It fails when

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you use broad tests to find successful techniques. The purpose of testing is to show robustness, that the method works over a wide range of situations in a similar manner. A robust solution will not appear to be as good as the optimal result, but performed properly, it will be a more realistic assessment of expectations.

Risk Control

Every system must control its risk, and most analysts believe that nearly any system can be profitable with proper risk management. This also means that any system can lead to ruin without risk controls. Risk can be managed from the trade level, with individual stoplosses, to asset allocation, by varying the size of the position traded, and by leveraging and deleveraging. Some form of management is necessary.

Order Entry

A system that performs well on paper may be dismal when actually traded. Part of a trading program is to know the method of entering and exiting the market and the cost of each trade. Style and cost will have a greater impact on short-term systems, which have a smaller profit per trade and are, therefore, more sensitive to transaction costs. There is equal damage in overestimating costs as there is in underestimating them. By burdening a system with unrealistic fees, it may show a loss when it should be a successful trading method.

Performance Monitoring and Feedback

A system is not done when you begin trading, it is only in a new phase. Actual trading results must be carefully monitored and compared with expectations to know if it is performing properly. It is very likely that slippage will result in some changes to the system rules or to the size of the position traded. Performance monitoring provides the essential feedback needed to be successful. Even a well thought-out and tested program may start out badly, but proper monitoring can put it on track.

A WORD ON NOTATION USED IN THIS BOOK

In attempting to make the contents of this book more practical for many readers, there are three types of

notation that can be found mixed together. Of course, the standard mathematical formulas for most methods appear as they had in the previous editions. Added to that are spreadsheet examples, using Corel's Quattro code, which is very similar to Microsoft's Excel. Readers should have no trouble transferring the examples found here to their own choice of spreadsheet.

Finally there is extensive program code with examples in Omega's Easy Language. Although these programs have been entered and tested on TradeStation, there are occasional errors introduced during final editing and in transferring the code into this book. Readers are advised to check over the code and test it thoroughly before using it. In addition, there are times when only a single line of code is shown along with the standard mathematical formula to help the reader translate the technique into a more practical form. Because of the many different forms of formulas, you may find that the standard deviation function takes the spreadsheet form of @std rather than the Easy Language notation @stddev, or that @avg appears instead of @average. Please check these formulas for notation consistent with your needs.

Basic Concepts

... economics is not an exact science: it consists merely, of Laws of Probability. The most prudent investor therefore, is one who pursues only a general course of action which is "normally" right and who avoids acts and policies which are---normally-wrong.

L.L.B. Angas

There will come a time when we no longer will know how to do the calculation for long division, because miniature voice-activated computers will be everywhere. We might not even need to be able to add; it will all be done for us. We might just assume that it is correct, because computers don't make mistakes. In a small way this is happening now. -Not everyone checks their more complicated spreadsheet calculations by- hand to be certain they are correct before going further. Nor does everyone print the intermediate results of computer calculations to verify their accuracy. Computers don't make mistakes. but people do.

With computer software rapidly making technical analysis easier, we no longer think of the steps involved in a moving average or linear regression. A few years ago -we used correlations only when absolutely necessary, because they were too complicated and time consuming to calculate. It would even be difficult to know if you had made a mistake without having someone else repeat the same calculations. -"sc" we face a different problem-if the computer does it all, we lose our understanding of why - 1, a moving average trendline differs from a linear regression. Without looking at the data, we don't see an erroneous outlier. By not reviewing each hypothetical trade, we miss seeing that the slip page can turn a profit into a loss.

To avoid losing the edge needed to create a profitable trading strategy. the basic tools of the trade are explained in this chapter. Those of you already familiar with these methods may skip over it; others should consider it essential that they be able to perform these calculations manually.

ABOUT DATA AND AVERAGING

The Law of Averages

The law of averages is a greatly misunderstood and misquoted principle. Its most often referred to when an abnormally long series of losses is expected to be off-set by an equal and opposite run of profits. It is equally wrong to expect a market that is currently overbought to next become oversold. That is not what is meant by the law of averages. Over a large sample, the bulk of events will be scattered close to the average in such a way as to overwhelm an abnormal set of events and cause them to be insignificant.

This principle is illustrated in Figure 2-1, where the addition of a small abnormal grouping to one side of a balanced group of near-normal data does not affect the balance. A long run of profits, losses, or price movement is simply abnormal and will be offset over

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time by the large number of normal events. Further discussion can be found in---"The Theory of Runs" (Chapter 22).

Bias in Data

When sampling is used to obtain data, it is common to divide entire subsets of data into discrete parts and attempt a representative sampling of each portion- These samples are then weighted to reflect the perceived impact of each part on the whole. Such a weighting will magnify or reduce the errors in each of the discrete sections. The result of such weighting may cause an error in bias. Even large numbers within a sample cannot overcome intentional bias introduced by weighting one or more parts

Price analysis and trading techniques often introduce bias in both implicit and explicit ways. A weighted average is an overt way of adding a positive bias (positive because it is intentional). On the other hand, the use of two analytic methods acting together may unknowingly rely doubly on one statistical aspect of the data; at the same time, other data may be used only once or may be eliminated by offsetting use. The daily high and low used in one part of a program and the daily range (high to low) in another section would introduce bias.

How Much Data Is Enough?

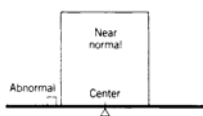
Technical analysis is fortunate to be based on a perfect set of data. Each price that is recorded by the exchange is exact and reflects the netting out of all information at that moment. Most other statistical data, although it might appear to be very specific, are normally an average value, which can represent a broad range of numbers, all of them either larger or smaller. The average price received by all farmers for corn on the 15th of the month cannot be the exact number. The price of Eurodollars at 10:05 in Chicago is the exact and only price.

When an average is used, it is necessary to collect enough data to make that average accurate. Because much statistical data is gathered by sampling, particular care is given to accumulating a sufficient amount of representative data. This will hold true with prices as well. Averaging a few prices, or analyzing small market moves, will show more erratic results. It is difficult to draw an accurate picture from a very small sample.

When using small, incomplete, or representative sets of data, the approximate error, or accuracy, of the sample should be known. This can be found by using the standard deviation as discussed in the previous section. A large standard deviation means an extremely scattered set of points, which in turn makes the average less representative of the data. This process is called the testing of significance. The most basic of these tests is the error resulting from a small amount of data. Accuracy usually increases as the number of items becomes larger, and the measurement of deviation or error will become proportionately smaller.

$$\text{Error} = \frac{1}{\sqrt{\text{number of items sampled}}} = \frac{1}{\sqrt{N}} \text{ or } 1/\sqrt{N}$$

FIGURE 2-1 The law of averages. The normal cases overwhelm the unusual ones. It is not necessary for the extreme cases to alternate—one higher, then the other lower—to create a balance.



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Therefore, using only one item has an error factor of 100%; with four items, the error is 50%. The size of the error is important to the reliability of any trading system. If a system has had only 4 trades, whether profits or losses, it is very difficult to draw any conclusions about performance expectations. There must be sufficient trades to assure a comfortably small error factor. To reduce the error to 5%, there must be 400 trades, which presents a dilemma for a very slow trend-following method that may only generate 2 or 3 trades each year. To compensate for this, the identical method can be applied to many markets and the sample of trades used collectively. By keeping the sample error small, the risk of trading can be better understood.

ON THE AVERAGE

In discussing numbers, it is often necessary to use representative values. The range of values or the average may be substituted to change a single price into a general characteristic to solve a problem. The average (arithmetic mean) of many values can be a preferable substitute for any one value. For example, the average retail price of one pound of coffee in the northeast is more meaningful to a cost-of-living calculation than the price at any one store. However, not all data can be combined or averaged and still have meaning. The average of all futures prices taken on the same day would not say anything about an individual market that was part of the average. The price changes in copper, corn, and the German DAX index, for example, would have little to do with one another. The average of a group of values must meaningfully represent the individual items.

The average can be misleading in other ways. Consider coffee, which rose from 40c to \$2.00 per pound in one year. The average price of this product may appear to be \$1.40; however, this would not account for the time that coffee spent at various price levels. Table 2-1 divides the coffee move into four equal price intervals, then shows that the time intervals spent at these levels were uniformly opposite to the price rise. That is, price remained at lower levels longer, and at higher levels for shorter time periods, which is very normal price behavior.

When the time spent at each price level is included, it can be seen that the average price should be lower than \$1.40. One way to calculate this, knowing the specific number of days in each interval, is by using a weighted average of the price and its respective interval.

$$W = \frac{a_1d_1 + a_2d_2 + a_3d_3 + a_4d_4}{d_1 + d_2 + d_3 + d_4}$$

$$W = 105$$

$$W = \frac{6000 + 8000 + 8400 + 7200}{280}$$

TABLE 2-1 Weighting an Average

Prices Go		Average during Interval	Total Days for Interval	Weighted	1/a
From	To				
40	80	$a_1 = 60$	$d_1 = 100$	6,000	.01666
80	120	$a_2 = 100$	$d_2 = 80$	8,000	.01000
120	160	$a_3 = 140$	$d_3 = 60$	8,400	.00714
160	200	$a_4 = 180$	$d_4 = 40$	7,200	.00555

Although this is not exact because of the use of average prices for intervals, it does closely represent the average price relative to time. There are two other averages for which time is an important element—the geometric and harmonic means.

Geometric Mean

The geometric mean represents a growth function in which a price change from 50 to 100 is as important as a change from 100 to 200.

$$G = (a_1 \times a_2 \times a_3 \times \dots \times a_n)^{1/n} \text{ or } @\text{mult}(\text{list})^{(1/\text{length of list})}$$

To solve this mathematically, rather than using a spreadsheet, the preceding equation can be changed to either of two forms:

$$\ln G = \frac{\ln a_1 + \ln a_2 + \dots + \ln a_n}{n}$$

or

$$\ln G = \frac{\ln (a_1 \times a_2 \times a_3 \times \dots \times a_n)}{n}$$

The two solutions are equivalent. Using the price levels in Table 2-1, disregarding the time intervals, and substituting into the first equation:

$$\ln G = \frac{\ln 40 + \ln 80 + \ln 120 + \ln 160 + \ln 200}{5}$$

Had one of the periods been a loss, that value would simply be negative. We now perform the arithmetic to solve the equation:

$$\ln G = \frac{3.689 + 4.382 + 4.787 + 5.075 + 5.298}{5}$$

$$\ln G = 4.6462$$

$$G = 104.19$$

The geometric mean has advantages in application to economics and prices. A classic example is to compare a tenfold rise in price from 100 to 1,000 with a fall to one-tenth from 100 to 10. An arithmetic mean of 10 and 1,000 is 505, while the geometric mean gives

$$G = (10 \times 1000)^{1/2} = 100$$

which shows the relative distribution as a function of comparable growth. Due to this property the geometric mean is the best choice when averaging ratios that can be either fractions or percentages.

Quadratic Mean

The quadratic mean is as calculated:

$$Q = \sqrt{\frac{\sum a^2}{N}} \text{ or } @\text{sqrt}(@\text{sum}(\text{list of } a^2) / \text{length of list})$$

The square root of the mean of the square of the items (root-mean-square) is most well known as the basis for the standard deviation. This will be discussed later, in the section "Dispersion and Skewness."

Harmonic Mean

The harmonic mean is more of a time-weighted average, not biased toward higher or lower values as in the geometric mean. A simple example is to consider the average speed of a car that travels 4 miles at 20 mph, then 4 miles

at 30 mph. An arithmetic mean would result in 25 mph, without considering that 12 minutes were spent at 20 mph and 8 minutes at 30 mph. The weighted average would give

$$W = \frac{(12 \times 20) + (8 \times 30)}{12 + 8} = 24$$

The harmonic mean is

$$\frac{1}{H} = \frac{\frac{1}{a_1} + \frac{1}{a_2} + \dots + \frac{1}{a_n}}{n}$$

which can also be expressed as

$$H = n / \sum_{i=1}^n \left(\frac{1}{a_i} \right) \text{ or length of list} / \text{sum(list of } 1/a)$$

For two or three elements, the simpler forms can be used:

$$H_2 = \frac{2ab}{a+b} \quad H_3 = \frac{3abc}{ab+ac+bc}$$

This allows the solution pattern to be seen. For the 20 and 30 mph rates of speed, the solution is

$$H_2 = \frac{2 \times 20 \times 30}{20 + 30} = 24$$

which is the same answer as the weighted average. Considering the original set of numbers again, the basic form of harmonic mean can be applied:

$$\begin{aligned} \frac{1}{H} &= \frac{\frac{1}{40} + \frac{1}{80} + \frac{1}{120} + \frac{1}{160} + \frac{1}{200}}{5} \\ &= \frac{.05708}{5} = .01142 \\ H &= 87.59 \end{aligned}$$

We might apply the harmonic mean to price swings, in which the first swing moved 20 points over 12 days, and the second swing moved 30 points over 8 days.

DISTRIBUTION

The measurement of distribution is very important because it tells you generally what to expect. We cannot know what tomorrow's S&P trading range will be, but we have a high level of confidence that it will fall between 300 and 800 points. We have a slightly lower confidence that it will vary from 400 to 600 points. We have virtually no chance of picking the exact range. The following measurements of distribution allow you to put a value on the chance of an event occurring.

Frequency Distributions

The frequency distribution can give a good picture of the characteristics of the data. To know how often sugar prices were at different price levels, divide prices into 10 increments (e.g., 5.01 to 6.00, 6.01 to 7.00, etc.), and count the number of times that prices fall into each interval. The result will be a distribution of prices as shown in Figure 2-2. It should be expected that the distribution of prices for a physical commodity interest rates (yield), or index markets, will be skewed toward the left-hand side (lower prices or yields) and have a long tail toward higher prices on the right-hand side. This is because prices remain at higher levels for only a short time relative to their long-term characteristics. Commodity prices tend to be bounded on the lower end, limited in their downside movement, by production costs and resistance of the suppliers to sell at prices that represent a loss. On the higher end, there is not such a clear point of limitation; therefore, prices move much further up during periods of extreme shortage relative to demand.

The measures of central tendency discussed in the previous section are used to qualify the shape and extremes of price movement shown in the frequency distribution. The general relationship between the results when using the three principal means is

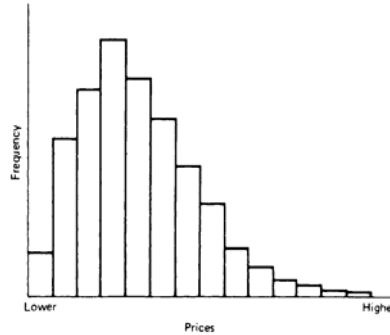
arithmetic mean > geometric mean > harmonic mean

Median and Mode

Two other measurements, the median and the mode, are often used to define distribution. The median, or middle item, is helpful for establishing the center of the data: it halves the number of data items. The median has the advantage of discounting extreme values, which might distort the arithmetic mean. The mode is the most commonly occurring value in Figure 2-3 the mode is the highest point.

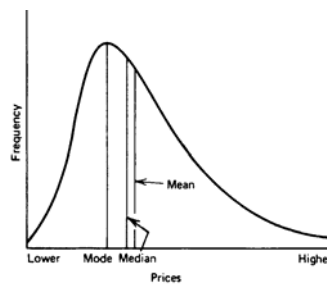
In a normally distributed price series, the mean, median, and mode will all occur at the same value; however, as the data become skewed, these values will move farther apart. The general relationship is:

FIGURE 2-2 Hypothetical price-frequency distribution.



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FIGURE 2-3 Hypothetical price distribution skewed to the right, showing the relationship of the mode, median, and mean.



mean > median > mode

The mean, median, and mode help to tell whether data is normally distributed or skewed. A normal distribution is commonly called a bell curve, and values fall equally on both sides of the mean. For much of the work done with price and performance data, the distributions tend to extend out toward the right (positive values) and be more cut off on the left (negative values). If you were to chart a distribution of trading profits and losses based on a trend system with a fixed stop-loss, you would get profits that could range from zero to very large values, while the losses would be theoretically limited to the size of the stop. Skewed distributions will be important when we try to measure the probabilities later in this chapter.

Characteristics of the Principal Averages

Each averaging method has its unique meaning and usefulness. The following summary points out their principal characteristics:

The arithmetic mean is affected by each data element equally, but it has a tendency to emphasize extreme values more than other methods. It is easily calculated and is subject to algebraic manipulation.

The geometric mean gives less weight to extreme variations than the arithmetic mean and is most important when using data representing ratios or rates of change. It cannot always be used for a combination of positive and negative numbers and is also subject to algebraic manipulation.

The harmonic mean is most applicable to time changes and, along with the geometric mean, has been used in economics for price analysis. The added complications of computation have caused this to be less popular than either of the other averages, although it is also capable of algebraic manipulation.

The mode is not affected by the size of the variations from the average, only the distribution. It is the location of

greatest concentration and indicates a typical value for a reasonably large sample. With an unordered set of data, the mode is time consuming to locate and is not capable of algebraic manipulation.

The median is most useful when the center of an incomplete set is needed. It is not affected by extreme variations and is simple to find if the number of data points are known. Although it has some arithmetic properties, it is not readily adaptable to computational methods.

DISPERSION AND SKEWNESS

The center or central tendency of a data series is not a sufficient description for price analysis. The manner in which it is scattered about a given point, its dispersion and shewness, are necessary to describe the data. The mean deviation is a basic method for measuring distribution and may be calculated about any measure of central location, for example, the arithmetic mean. It is found by computing

$$MD = \frac{\sum |price_i - \frac{\sum price_i}{n}|}{n} \quad \text{or} \quad @Avg(@AbsValue(price - @Avg(price,n)),n)$$

where MD is the mean deviation, the average of the differences between each price and the arithmetic mean of the prices, or other measure of central location, with signs ignored.

The standard deviation is a special form of measuring average deviation from the mean, which uses the root-mean-square

$$\text{Standard deviation} = \sqrt{\frac{\sum (price_i - \frac{\sum price_i}{n})^2}{n}} \quad \text{or} \quad = @Sqrt(@Avg((price - @Avg(price,n))^2),n)$$

where the differences between the individual prices and the mean are squared to emphasize the significance of extreme values, and then total final value is scaled back using the square root function. This popular measure, found throughout this book, is available in all spreadsheets and software programs as @Std or @Stdey For n prices, the standard deviation is simply @Std(price,n).

The standard deviation is the most popular way of measuring the degree of dispersion of the data. The value of one standard deviation about the mean represents a clustering of about 68% of the data, two standard deviations from the mean include 95.5% of all data, and three standard deviations encompass 99.7%, nearly all the data. These values represent the groupings of a perfectly normal set of data, shown in Figure 2-4.

Probability of Achieving a Return

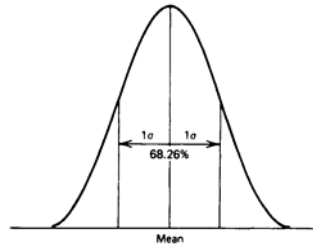
If we look at Figure 2-4 as the annual returns for the stock market over the past 50 years, then the mean is about 8% and one standard deviation is 16%. In any one year we can expect the compounded rate of return to be 8%; however, there is a 32% chance that it will be either greater than 24% (mean plus one standard deviation) or less than -8% (the mean minus one standard deviation). If you would like to know the probability of a return of 20% or greater, you must first rescale the values,

$$\text{Probability of reaching objective} = \frac{\text{objective} - \text{mean}}{\text{standard deviation}}$$

If your objective is 20%, we calculate

$$\text{Probability} = \frac{20\% - 8\%}{16\%} = .75$$

FIGURE 2-4 Normal distribution showing the percentage area included within one standard deviation about the arithmetic mean.



We look in Appendix A1 under the probability for normal curves, and find that a standard deviation of .75 gives 27.34%, a grouping of 54.68% of the data. That leaves one-half of the remaining data, or 22.66%, above the target of 20%.

Skewness

Most price data, however, are not normally distributed. For physical commodities, such as gold, grains, and interest rates (yield), prices tend to spend more time at low levels and much less time at extreme highs; while gold peaked at \$800 per ounce for one day, it has remained between \$375 and \$400 per ounce for most of the past 10 years. The possibility of failing below \$400 by the same amount as its rise to \$800 is impossible, unless you believe that gold can go to zero. This relationship of price versus time, in which markets spend more time at lower levels, can be measured as skewness—the amount of distortion from a symmetric shape that makes the curve appear to be short on one side and extended on the other. In a perfectly normal distribution, the median and mode coincide. As prices become extremely high, which often happens for short intervals of time, the mean will show the greatest change and the mode will show the least. The difference between the mean and the mode, adjusted for dispersion using the standard deviation of the distribution, gives a good measure of skewness

$$(\text{Skewness}) S_k = \frac{\text{mean} - \text{mode}}{\text{standard deviation}}$$

Because the distance between the mean and the mode, in a moderately skewed distribution, is three times the distance between the mean and the median, the relationship can also be written as:

$$(\text{Skewness}) S_k = \frac{3 \times (\text{mean} - \text{median})}{\text{standard deviation}}$$

This last formula may be more practical for computer applications, because the mode requires dividing the data into groups and counting the number of occurrences in each bar. When interpreting the value of S_k , the distribution leans to the right when S_k is positive (the mean is greater than the median), and it is skewed left when S_k is negative.

Kurtosis

One last measurement, that of kurtosis, should be familiar to analysts. Kurtosis is the "peakedness" of a distribution, the analysis of "central tendency." For most cases a smaller standard deviation means that prices are clustered closer together; however, this does not always describe the distribution clearly. Because so much of identifying a trend comes down to deciding whether a price change is normal or likely to be a leading indicator of a new direction, deciding whether prices are closely grouped or broadly distributed may be useful. Kurtosis measures the height of the distribution.

Transformations

The skewness of a data series can sometimes be corrected using a transformation on the data. Price data may be skewed in a specific pattern. For example, if there are 1/4 of the occurrences at twice the price and 1/9 of the occurrences at three times the price, the original data can be transformed into a normal distribution by taking the square root of each data item. The characteristics of price data often show a logarithmic, power, or square-root relationship.

Skewness in Price Distributions

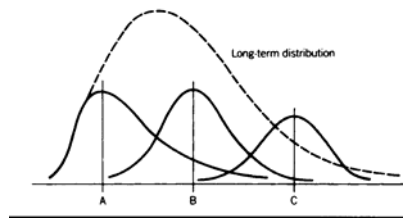
Because the lower price levels of most commodities are determined by production costs, price distributions show a clear boundary of resistance in that direction. At the high levels, prices can have a very long tail of low

frequency Figure 2-5 shows the change in the distribution of prices as the mean price (over shorter intervals) changes. This pattern indicates that a normal distribution is not appropriate for commodity prices, and that a log distribution would only apply to overall long-term distributions.

Choosing between Frequency Distribution and Standard Deviation

You should note that it is more likely that unreliable probabilities will result from using too little data than from the choice of method. For example, we might choose to look at the distribution of one month of daily data, about 23 days; however, it is not much of a sample. The price or equity changes being measured might be completely different during the next month. Even the most recent five years of S&P data will not show a drop as large as October 1987.

FIGURE 2-5 Changing distribution at different price levels. A, B, and C are increasing mean values of three shorter-term distributions.



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Although we can identify and measure skewness, it is difficult to get meaningful probabilities using a standard deviation taken on very distorted distributions. It is simpler to use a frequency distribution for data with long tails on one side and truncated results on the other. To find the likelihood of returns using a trend system with a stop-loss, you can simply sort the data in ascending order using a spreadsheet, then count from each end to find the extremes. You will notice that the largest 10% of the profits cover a wide range, while the largest 10% of the losses is clustered together.

A standard deviation is very helpful for giving some indication that a price move, larger than any we have seen in the data, is possible. Because it assumes a normally shaped curve, a large clustering of data toward one end will force the curve to extend further. Although the usefulness of the exact probabilities is questionable, there is no doubt that, given enough time, we will see price moves, profits, and losses that are larger than we have seen in the past.

Student t-test

Throughout the development and testing of a trading system, we want to know if the results we are seeing are as expected. The answer will keep referring back to the size of the sample and the amount of variance that is typical of the data during this period. Readers are encouraged to refer to other sections in the book on sample error and chi-square test. Another popular method for measuring whether the average price of the data is significantly different from zero, that is, if there is an underlying trend bias or if the pattern exhibits random qualities, is the student t-test,

$$t = \frac{\text{average of price changes}}{\text{standard deviation of price changes}} \times \sqrt{\text{No. of data items}}$$

and where degrees of freedom = number of data items - 1. The more trades in the sample, the more reliable the results. The values of t needed to be significant can be found in Appendix 1, Table A1.2, "T-Distribution." The column headed ".10" gives the 90% confidence level, ".05" is 95%, and ".005" is 99.5% confidence.

If we separate data into two periods and compare the average of the two periods for consistency, we can decide whether the data has changed significantly. This is done with a 2-sample t-test:

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{v_1}{n_1} + \frac{v_2}{n_2}}}$$

where \bar{x}_1 and \bar{x}_2 are the averages of data periods 1 and 2, v_1 and v_2 are the variances of periods 1 and 2, and n_1 and n_2 are the number of data items in periods 1 and 2. The degrees of freedom, *df*, needed to find the confidence levels in Table A1-2, can be calculated as:

$$df = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2} \right)^2}{\frac{\left(\frac{s_1^2}{n_1} \right)^2}{(n_1 - n_2)} + \frac{\left(\frac{s_2^2}{n_2} \right)^2}{(n_2 - n_1)}}$$

The student t-test can also be used to compare the profits and losses generated by a trading system to show that the underlying system process is sound. Simply replace the data items by the average profit or loss of the system, the number of data items by the number

of trades, and calculate all other values using the profit/loss to get the student t-test value for the trading performance.

STANDARDIZING RETURNS AND RISK

To compare one trading method with another, it is necessary to standardize both the tests and the measurements used for evaluation. If one system has total returns of 50% and the other of 250%, we cannot decide which is best unless we know the duration of the test. If the 50% return was over 1 year and the 250% return over 10 years, then the first one is best. Similarly, the return relative to the risk is crucial to performance as will be discussed in Chapter 21 ("Testing"). For now it is only important that returns and risk be annualized or standardized to make comparisons valid.

Calculating Returns

The calculation of rate of return is essential for assessing performance as well as for many arbitrage situations. In its simplest form, the one-period rate of return R, or the holding period rate of return is

$$\text{Return} = \frac{\text{Ending value} - \text{Starting value}}{\text{Starting value}} = \frac{\text{Ending value}}{\text{Starting value}} - 1$$

$$R = \frac{P_1 - P_0}{P_0} = \frac{P_1}{P_0} - 1$$

where P_0 is the initial investment or starting value, and P_1 is the value of the investment after one period. In most cases, it is desirable to standardize the returns by annualizing. This is particularly helpful when comparing two sets of test results, in which each covers a different time period. Although calculations on government instruments use a 360-day rate (based on 90-day quarters), a 365-day rate is common for most other purposes. The following formulas show 365 days; however, 360 may be substituted.

The annualized rate of return on a simple-interest basis for an investment over n days is

$$R_{\text{simple}} = \frac{P_1 - P_0}{P_0} \times \frac{365}{n}$$

The *annualized compounded rate of return* is

$$R_{\text{compounded}} = \left(1 + \frac{P_n - P_0}{P_0} \right)^{(365/n)}$$

The geometric mean is the basis for the compounded growth associated with interest rates. If the initial investment is \$1,000 (P_0) and the ending value is \$1,600 (P_y) after 12 years ($y = 12$), there has been an increase of 60%. The simple rate of return is 5%, but the compounded growth shows

$$\text{Ending value} = \text{Starting value} \times (1 + \text{Compounded return})^y$$

$$P_y = P_0 \times (1 + R)^y$$

or

$$R = \left(\frac{P_y}{P_0} \right)^{(1/y)} - 1$$

$$R = \left(\frac{1600}{1000} \right)^{(1/12)} - 1$$

$$R = .0398 \text{ or about } 4.0\%$$

The use of the standard deviation and compounded rate of return are combined to find the probability of a return objective. In the following calculation,¹ the arithmetic mean of continuous returns is $\ln(1 + R_g)$, and it is assumed that the returns are normally distributed.

$$z = \frac{\ln\left(\frac{T}{B}\right) - \ln(1 + R_g)n}{\sqrt{s \times n}}$$

where z = standardized variable (can be looked up in Appendix A1)

T = target value or rate-of-return objective

B = beginning investment value

R_g = geometric average of periodic returns

n = number of periods

s = standard deviation of the logarithms of the quantities 1 plus the periodic returns

Indexing Returns

The Federal Government has defined standards for calculating returns in the Futures Industry Commodity Trading Advisors (CTAs). This is simply an indexing of returns based on the current period percentage change in equity. It is the same process as creating any index, and it allows trading returns to be compared with, for example, the S&P Index or the Lehman Brothers Treasury Index, on equal footing. Readers should refer to the section later in this chapter, "Constructing an Index."

Calculating Risk

Although we would always like to think about returns, it is even more important to be able to assess risk. With that in mind, there are two types of risk that are important for very different reasons. The first is catastrophic risk, which will cause fatal losses or ruin. This is a complicated type of risk, because it may be the result of a single price shock or a steady deterioration of equity by being overleveraged. This form of risk will be discussed in detail later in the book.

Standard risk measurements are useful for comparing the performance of two systems and for understanding how someone else might evaluate your own equity profile. The simplest estimate of risk is the variance of equity over a time interval commonly used by most investment managers. To calculate the variance, it is first necessary to find the mean return, or the expected return, on an investment:

$$E(R) = p_1R_1 + p_2R_2 + \dots + p_nR_n$$

The most common measure of risk is variance, calculated by squaring the deviation of each return from the mean, then multiplying each value by its associated probability

$$\text{variance} = \sum_{i=1}^n p_i R_i^2$$

The sum of these values is called the variance, and the square root of the variance is called the standard deviation. This was given in another form in the early section "Dispersion and Skewness."

$$\begin{aligned} \text{standard deviation} &= \sqrt{p_1[R_1 - E(R)]^2 + p_2[R_2 - E(R)]^2 + \dots + p_n[R_n - E(R)]^2} \\ &= \sqrt{\sum_{i=1}^n p_i [R_i - E(R)]^2} \end{aligned}$$

¹This and other very clear explanations of returns can be found in Peter L. Bernstein's *The Portable MBA in Investment* (John Wiley & Sons, New York, 1995).

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